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**Theory and Construction Methods for
Large Regular Resolution IV Designs**

A Dissertation

Presented for the

Doctor of Philosophy

Degree

University of Tennessee, Knoxville

Robert M. Block

August 2003

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Dedication

To my family, thank you for all the love and support.

Acknowledgements

I wish to express my deepest gratitude and thanks to my advisor, Dr Robert Mee. I have cherished the many hours spent in his office discussing not only designs of experiments, but life's challenges as well. Without his help, this work would not have been accomplished.

I would also like to thank the other members of my dissertation committee. I have enjoyed my hours in and out of the classroom with Dr Mary Leitnaker; who always gave me good advice and lots of support. I want to thank Dr Ken Kirby for showing me the connection between lean production and statistics and letting me seek his advice both about statistics and help for practical matters as well. I would also like to thank Dr Bill Parr for his help and work as a committee member.

Abstract

We define 2^{k-p} fractional factorial designs which use all of their degrees of freedom to estimate main effects and two-factor interactions as *second order saturated* (sos) designs. We prove that resolution IV sos designs project to every other resolution IV design, and show the details of these projections for every $n = 32$ and $n = 64$ run fraction. For $k > (5/16)n$, all resolution IV designs are a projection from the even sos design at $k = n/2$. For $k \leq (5/16)n$ the minimum aberration design resolution IV designs are projections of sos designs with both even and odd words in the defining relation. While even resolution IV designs are limited to estimating fewer than $n/2$ two-factor interactions (in addition to the k main effects), resolution IV designs with odd-length words in the defining relation may devote more than half of their degrees of freedom to two-factor interactions. We propose a method to search for good resolution IV designs using naïve projections from even/odd sos designs. We introduce the alias length pattern as a tool to help characterize designs. We describe how the matrix $T = DD'$ for a design D is useful in searching for designs. We list the resolution IV even/odd minimum aberration designs for $n = 128$ and provide a catalog of the best resolution IV even/odd designs for $n = 128$. These results are based on an isomorphic check using a convenient function of T , as well as the set of projections of a design. Finally, we suggest a new method for finding good regular resolution IV designs for large n (> 128) and provide a preliminary table of good resolution IV even/odd designs for $n = 256$.

Key words: alias length pattern, defining contrast subgroup, Hamming distance matrix, isomorphism, minimum aberration, projection, regular designs, word length pattern.

Disclaimer

The views expressed in this dissertation are those of the author and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the U.S. Government.

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1. Introduction

Two-level fractional factorial designs are widely used to investigate the effect of large numbers of parameters for complex computer models. Each parameter is varied over a high and low setting of possible operating conditions to build a model to help explain the relationship of the parameters to the outcome of the computer model. A 2^{k-p} fractional factorial design with k parameters or factors at two levels will consist of $n = 2^{k-p}$ runs. This design is a 2^{-p} th fraction of the 2^k full factorial design where the fraction is determined by p defining words. A "word" consists of "letters" which are the names of the factors denoted by A, B, ... (or 1, 2, ...). The number of letters in a word is the word length. The group formed by the p defining words and their generalized interactions is called the defining contrast subgroup (Wu and Hamada 2000, p.157). The defining contrast subgroup consists of $2^p - 1$ words plus the identity column (commonly denoted as I). The defining contrast subgroup can be used to study all the aliasing relations among effects.

Every regular design can be categorized by the word length pattern of its defining contrast subgroup. For a 2^{k-p} design, let w_i denote the number of words of length i in its defining contrast subgroup. The vector $wlp = (w_1, \dots, w_k)$ is called the word length pattern of the design. The resolution of a 2^{k-p} design is defined to be the smallest r such that $w_r \geq 1$. This means the length of the shortest word defines the resolution. Box and Hunter (1961) proposed the maximum resolution criterion as a method to categorize and compare designs. Later, Fries and Hunter (1980) introduced the minimum aberration criteria. This criterion allows any two designs to be rank ordered according to their word

length patterns. This is the most common criterion used today to judge the goodness of designs.

In addition to wlp, we introduce a new criterion based on the alias length pattern to help find and characterize resolution IV designs. We define the alias length pattern as the frequencies of the lengths of the alias sets for two-factor interactions:

$alp = (a_1, a_2, \dots, a_l)$ where a_1 is the number of clear two-factor interactions, a_2 is the number of pairs of aliased two-factor interactions, etc., up to a_l which is the number of the largest set of l aliased two-factor interactions $\left(l \leq \left\lfloor \frac{k}{2} \right\rfloor \right)$, we define this value as L_{\max} .

The alias length pattern (alp) also contains other important information:

- The number of degrees of freedom for two-factor interactions: $\sum_{i=1}^l a_i$
- The number of length four words in the defining relation: $w_4 = \sum_{i=2}^l \binom{i}{2} a_i / 3$.

All regular 2_{IV}^{k-p} designs of size $n = 64$ or less have been identified previously; see Chen, Sun and Wu (CSW) (1993) and Sun (2001). However, for $n = 128$, all possible resolution IV designs have not been identified. Butler (2003) provided theory for constructing regular minimum aberration designs with n runs and $5n/16 < k < n$ factors. We have identified all remaining minimum aberration designs for $n = 128$, that is, for $k \leq 5n/16$.

For cases with $n = 128$ or more, search algorithms are currently used to identify attractive fractional factorial designs having the specified size and other characteristics.

For example, PROC FACTEX in SAS/QC[®] software (SAS Institute Inc., 1999) searches for minimum aberration designs for any given $k < 2^r$. However, due to the magnitude of the computation for large n and certain values of k , exhaustive searches are not feasible given current computing speeds. The FACTEX procedure returns the best design it finds in the allotted search time. It does not necessarily find the minimum aberration design. This paper will propose an alternative search method for tabulating good designs for $n = 256$ and larger.

It is well known that, for $k \leq n/2$ factors and $n = 8, 16, 24, 32, \dots$, there exist resolution IV designs. When $k = n/2$, the design is known as a *minimal design* of resolution IV (Montgomery 2001, p. 347). These minimal designs may be obtained by foldover of a saturated orthogonal main effects design of size $n/2$. For any $n = 2^r$ (with $r \geq 3$), a regular minimal design may be constructed by using all the odd interactions of the r basic columns as generators. For example, for $r = 5$, the 11 generators for the 2_{IV}^{16-11} design are the $\binom{5}{3} = 10$ three-factor interactions and the single five-factor interaction.

Alternatively one may arrange the $n - 1$ columns of a saturated main effects design in Yates order (e.g., see Appendix A), and:

- select every other column starting with the first or
- select the last $n/2$ columns.

Li and Mee (2002) present an alternative set of $n/2$ columns to create this minimal design.

For the remainder of this article, we restrict our attention to regular resolution IV designs.

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The minimal 2_{IV}^{k-p} designs are even designs, in that every word in the defining relation is of even length. Li and Mee (2002) showed that every 2_{IV}^{k-p} design with $5n/16 < k \leq n/2$ must be an even design. Even designs:

- alias even effects with other even effects, and odd effects with odd.
- allocate $n/2$ degrees of freedom to odd effects, and $n/2 - 1$ degrees of freedom to even effects
- provide at most $n/2 - 1$ degrees of freedom for estimating two-factor interactions, and at least $n/2 - k$ degrees of freedom for three-factor or higher-order interactions.

For instance, the minimum aberration 2_{IV}^{11-6} design - an even design - permits estimation of 11 main effects, 15 two-factor interactions, while leaving five degrees of freedom for aliased three-factor interactions.

By contrast, 2_{IV}^{k-p} designs with half of the words in the defining relation with odd length may provide more than $n/2 - 1$ degrees of freedom for two-factor interactions. For instance, the minimum aberration 2_{IV}^{10-5} design supports estimation of all 10 main effects and 21 two-factor interactions. Because of this greater capacity for estimating two-factor interactions, this work will focus on the construction of even/odd 2_{IV}^{k-p} designs. While such designs do not exist for $n = 16$ and are rather rare for $n = 32$, even/odd designs are common for larger n if $k \leq 5n/16$.

One of the challenging aspects of searching for new designs is determining when two designs are equivalent or isomorphic. (Two designs are isomorphic if the defining relation of one can be mapped into the defining relation of the other through a relabeling

of the factors and level exchanges.) Draper and Mitchell (1967, 1968, 1970) wrote a series of three articles which used an algorithm to determine isomorphic designs. Their original method, called "sequential conjecture" (1967) found a relabeling map for isomorphic designs. They noted in their next paper (1968) that word length pattern did not uniquely identify designs but it provided an alternative to their permutation subroutine (sequential conjecture procedure) for testing isomorphic designs when the time required to conduct the isomorphic checks become prohibitive. The trade-off of using word length pattern is that the designs found may not be a complete set. Draper and Mitchell (1970) introduced the "letter pattern comparison" (now commonly known as the letter pattern matrix) as a way to identify designs instead of the computationally burdensome sequential conjecture procedure. They make the conjecture that the letter pattern matrix approach uniquely determines designs. Chen and Lin (1991) provide a counter-example to this conjecture. Additional counter-examples appear later in section 11 in this dissertation.

Chen, Sun, and Wu (1993) developed an algorithm for constructing regular fractional factorial designs that required a complete mapping for each design that shared word length pattern. This method insured that no non-isomorphic designs were lost, but became computationally infeasible for $n = 128$ or larger.

Sun, Li, and Ye (2002) proposed a sequential method for constructing non-isomorphic orthogonal designs and an algorithm for detecting isomorphic designs for both regular and non-regular designs. Their algorithm is based on the concept of *minimal column base*. A column base is a subset of columns of a design, such that no two rows are identical to each other. A minimal column base is the smallest possible number of

columns for a given design. Sun, Li, and Ye check the mapping for the minimal column bases for two designs with the same word length pattern. They repeat this until an isomorphic mapping is found or all the possible minimal bases for the two designs have been checked. See Sun, Li, and Ye (2002) for details. This method is successful for both regular and non-regular designs and especially useful for designs with small n .

In the following section, we focus on the structure of even/odd resolution IV designs of size 32 and 64. We use these known cases to introduce some definitions and indicate the structure one could exploit in the larger cases where all designs are not known.

2. Resolution IV Designs of Size 32 and 64

Only five even/odd 2_{IV}^{k-p} designs of size 32 exist; refer to Table 2.1. For convenience, we use Chen, Sun, and Wu's method of labeling designs where 10-5.1 designates the first (best) 32 run design with ten factors and five generators. Two of these designs (10-5.1 and 9-4.2) utilize all 31 degrees of freedom for estimating main effects and two-factor interactions. We will refer to any 2_{IV}^{k-p} design (both even and even/odd designs) that uses all of its degrees of freedom for estimating main effects and two-factor interactions as a *second order saturated (sos) design*. Each of the non-sos designs is a projection of at least one of these sos designs. For instance, delete any column from 10-5.1 and one obtains design 9-4.1.

Theorem 2.1: Every 2_{IV}^{k-p} non-sos resolution IV design is the projection of at least one sos resolution IV parent design.

Suppose there exists a 2_{IV}^{k-p} non-sos design. A non-sos design is defined as a design that does not utilize all $2^{k-p} - 1$ degrees of freedom for estimating main effects and two-factor interactions.

Table 2.1: Even-Odd Resolution IV Designs of Size 32

Design	Generators	df	wlp	alp	E/O Projections
10-5.1	7, 11, 19, 29, 30	31	10,16,0,0,5	0,20,0,0,1	9-4.1
9-4.1	7, 11, 29, 30	30	6,8,0,0,1	8,12,0,1	8-3.1
9-4.2	7, 11, 13, 30	31	7,7,0,0,0,1	15,0,7	8-3.1
8-3.1	7,11,29	29	3,4	13,6,1	7-2.1
7-2.1	7, 27	25	1,2	15,3	

A non-sos design therefore has "available columns" for the unused degrees of freedom. An available column is any column that is not aliased with a main effect or two-factor interaction.

Suppose we add a new factor to our design, with an available column as its generator. The new factor "z" multiplied by its generator will appear as an additional word in the defining contrast subgroup. The new word is necessarily of length four or more and the resulting design with $k + 1$ factors must be resolution IV for the reason given below.

Suppose it is not resolution IV; then this would mean there is a word in the defining contrast subgroup of length three or less. This implies that a new word contains z (since z appears in all the new words) plus two or fewer other letters. This implies that z is aliased with either a main effect or two-factor interaction, which contradicts the fact that the generator was an "available column". Therefore the resulting $k + 1$ factor design must be resolution IV.

Now this $k + 1$ factor resolution IV design is either a second order saturated design with no more available columns, or a non-sos design with an available column. If not sos, the process can be repeated until the design becomes a second order saturated design. Therefore, all non-sos 2_{IV}^{k-p} designs have at least one resolution IV sos parent.

Corollary 2.1: All non-sos even/odd resolution IV designs are the projection of an even/odd resolution IV sos design.

Even/odd designs may project to either an even design or an even/odd design while even designs only project to other even designs (see Figure 2.1).

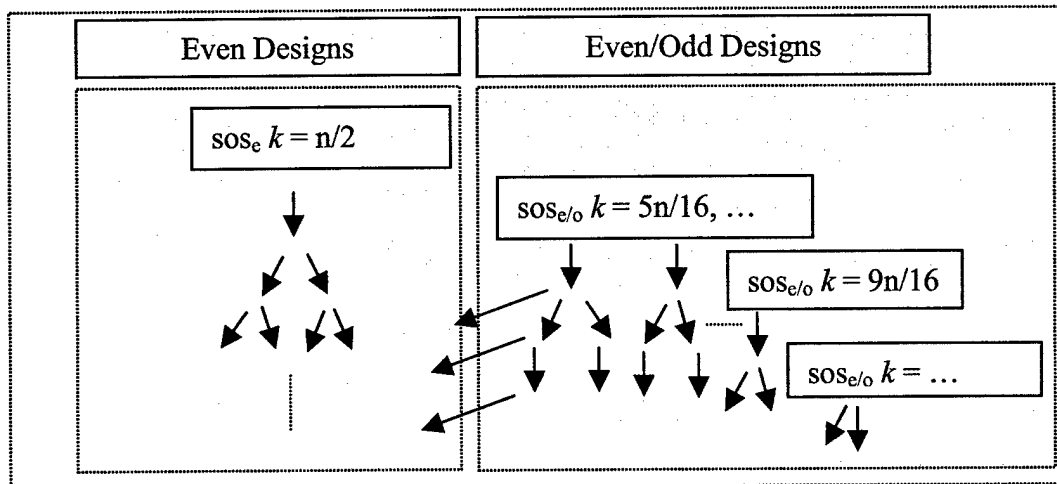


Figure 2.1: Schematic of Projections

Lemma 2.1: If the delete-one-column projections of an even/odd resolution IV design include multiple even designs, the even designs must be isomorphic.

We know that an even design will have all even length words in the defining relation while an even/odd design has 2^{p-1} odd-length words and $2^{p-1} - 1$ even-length words. If an even/odd design projects to an even design, then all the odd length words have been removed. Note that the projected even design may be written as a $2^{(k-1)-(p-1)}$; so half of the words in the defining relation have been removed. Therefore all the odd length words must contain the deleted column. Any other even projection must be isomorphic.

Table 2.1 includes the generators, degrees of freedom (for main effects and two-factor interactions), word length pattern (wlp) and the alias length pattern (alp) for each of the 32-run even/odd designs. For example, design 9-4.1 has $a_1 = 8$ clear two-factor interactions, $a_2 = 12$ pairs of aliased two-factor interactions, $a_4 = 1$ set of four aliased two-factor interactions, and $9 + 21 = 30$ degrees of freedom for main effects and two-factor interactions.

The catalog of designs in Appendix B shows all 148 even/odd 2_{IV}^{k-p} designs of size 64. Here we use our own notation to identify the designs since CSW (1993) did not list all the $n = 64$ designs in their catalog and their ordering did not accord with any obvious criteria. We rank the alternative 2_{IV}^{k-p} designs for a given k using the following criterion:

1. Smaller w_4
2. For designs with the same w_4 , smaller w_5
3. For designs with the same (w_4, w_5) , larger a_1

To avoid confusion with the CSW numbering, we use the letters a, b, ... rather than numerals to index the designs. Table B.1 does include a column identifying the CSW number for those designs that are included in their 1993 catalog.

We make the following observations regarding the catalog in Appendix B. First, there are only eight even/odd second order saturated resolution IV designs of size 64:

- 20-14.a
- 18-12.c
- 17-11.b,d,e,g,j
- 13-7.b

Second, a non-sos design in Appendix B may be the projection of more than one sos design. For instance, 16-10.b is the projection of either sos design 17-11.b or 17-11.d.

Note that each $n = 8, 16, 32, \dots$ there is only one even resolution IV second-order saturated (sos) design, the minimal design with $k = n/2$. Thus, the following results are apparent:

- For $n = 8$ and 16 , there exists only the unique even sos design with $k = n/2$.
- For $n = 32$, there exist three sos designs, with $k = 9, 10$, and 16 .
- For $n = 64$, there exist nine sos designs, with $k = 13, 17, 18, 20$, and 32 .

The sos designs with the smallest k are of particular interest because these designs provide the most degrees of freedom for two-factor interactions. We examine the 9-4.2 and 13-7.b designs now. Design 9-4.2 has $w_4 = 7$, and these length-four words involve only seven of the nine factors. Thus, all the interactions involving two factors are clear.

This design is structured as $\frac{1}{2} \left[2_{IV}^{7-3} \times 2^2 \right]$, where the one-half fraction of the product array is obtained by dividing each smaller design into two blocks and then taking only two of the four block combinations (see Figure 2.2) where the 2_{IV}^{7-3} has generators $6 = 123, 7 = 124, 8 = 134$. Note that the product array above is fractionated using $I = +23459$.

Design 13-7.b has similar structure: $\frac{1}{4} \left[2_{IV}^{7-3} \times 2_{IV}^{6-2} \right]$, with each 16-run sub-design divided into four blocks (see Figure 2.3). Butler (2002a) describes these types of designs as joint designs; see also Miller (1997).

		2^2	
		I = -59 (2 runs)	I = 59 (2 runs)
2_{IV}^{7-3} with	I = -234 (8 runs)	8x2=16 runs	
	I = 234 (8 runs)		8x2=16 runs

Figure 2.2: Design Structure for 9-4.2

		2_{IV}^{6-2} with <u>11</u> = 56 <u>10</u> and <u>13</u> = 56 <u>12</u>			
		<u>611</u> = + <u>61013</u> = +	<u>611</u> = + <u>61013</u> = -	<u>611</u> = - <u>61013</u> = +	<u>611</u> = - <u>61013</u> = -
2_{IV}^{7-3} with 7=123, 8=124, 9=134	1 = + 234 = +	4x4 = 16 runs			
	1 = + 234 = -		4x4 = 16 runs		
	1 = - 234 = +			4x4 = 16 runs	
	1 = - 234 = -				4x4 = 16 runs

Figure 2.3: Design Structure for 13-7.b

3. Projection Design Search Method

The difficulty of finding minimum aberration designs (and other good designs) increases dramatically as the size of the designs grows. As n becomes larger, it is no longer feasible to conduct exhaustive searches. One option is to intelligently reduce the number of designs that must be investigated. The value of sos designs is they represent a small fraction of all possible resolution IV designs and project to all the remaining possible resolution IV designs. Thus from these designs one can project to minimum aberration and other good designs. If all the sos designs for a given n can be found and identified, then we have the starting points for all resolution IV even or even/odd designs for a given n .

Our first attempt to find minimum aberration and other good designs was to find all the sos designs for a given run size n and then project from those designs to identify the best designs. To accomplish this requires the ability to find sos designs, distinguish non-isomorphic sos designs, and then to determine the best projections.

The first issue is feasible at $n = 128$. It appears to be possible to find the sos designs at $n = 128$. Projections of these sos designs lead to weak minimum aberration designs and careful evaluation of all sos designs would determine minimum aberration for any $k \leq 64$ at $n = 128$. There are 88 unique sos designs at $n = 128$. However to find the minimum aberration design, one must evaluate all possible sequence of projections; this combinatorial problem currently becomes computationally infeasible beyond ten or more projections. Therefore the projection search method is limited in its usefulness for conducting an exhaustive search; in addition, the number of sos designs explodes at higher n . For instance, there are at least 34,015 (and possibly twice that many) sos

designs at $n = 256$ (see section 13). Thus we found it necessary to pursue alternative methods.

4. Detecting Isomorphic Designs

To successfully find minimum aberration designs requires a computationally fast and efficient method to find and compare designs, as well as some ability to quickly identify isomorphic designs.

When searching for designs, most of the time is spent evaluating isomorphic designs. CSW (1993) were not able to distinguish all $n = 128$ designs beyond $k = 11$ because of the time required to find a complete relabeling of columns for every isomorphic design check. At $n = 128$ with $k = 11$ factors, there are 2,597 sets of four generators that produce a resolution IV designs. Of these designs, there are only 92 non-isomorphic designs. This is the last step CSW completed (Sun 2001). Consider at $k = 17$ we have found 14,438 unique resolution IV designs, and a total of 302,384 sets of ten generators producing a resolution IV design. Thus, on average, there are more than 20 ways to construct each unique design and the number of designs to compare is two orders of magnitude greater.

Two fractional factorial designs are isomorphic ($D_1 \cong D_2$) if one design can be obtained from the other design by relabeling the factors, reordering the runs, or switching the levels of factors (Chen and Lin 1991). Clark and Dean (2001) present a necessary and sufficient condition for two designs to be isomorphic based on a geometrical representation of the designs. Let D represent an $n \times k$ design matrix with n runs, k factors, and levels ± 1 . Let $T(D) = DD'$, which is related to the Hamming distance matrix H , since $T = kJ_k - 2H$ where J_k is a $k \times k$ matrix of unit elements. Note that for any design D , the $(i, j)^{\text{th}}$ element of T , denoted as $T_{ij}(D)$, is equal to the inner product of

the i^{th} and j^{th} rows of D . Clearly $T_{ij}(D) = k$ for $i = j$. Other properties of T are discussed in sections five and six. We now describe a result from Clark and Dean (2001) and introduce more notation:

Clark and Dean's Corollary 2.2: Designs D_1 and D_2 are isomorphic if and only if there exists an $n \times n$ permutation matrix R and a permutation $\{c_1, c_2, \dots, c_k\}$ of $\{1, 2, \dots, k\}$ such that, for $q = 1, 2, \dots, k$: $T(D_1^{\{1, 2, \dots, q\}}) = RT(D_2^{\{c_1, c_2, \dots, c_q\}})R'$ where $D^{\{1, 2, \dots, q\}}$ denotes a q -factor subset of the full design including just the listed columns.

We will say that $T(D_1)$ is equivalent to $T(D_2)$ [denoted as $T(D_1) \equiv T(D_2)$] if for some permutation matrix R , $T(D_1) = RT(D_2)R'$. Define $D_i^{\{q\}}$ to represent the design with only the q^{th} column from D_i . Similarly, $D_i^{\{\bar{q}\}}$ is the design matrix with all the columns of D_i except for column q . Observe that $T_{ij}(D^{\{\bar{q}\}}) = (k - 1)$ for $i = j$. Based on Clark and Dean's Corollary, we have Lemma 4.1:

Lemma 4.1: $D_1 \cong D_2$ if and only if $T(D_1) \equiv T(D_2)$ and $D_1^{\{\bar{q}\}} \cong D_2^{\{\bar{c}_q\}}$ for some integers q and c_q .

Note that by Clark and Dean's Corollary 2.2 $D_1^{\{\bar{k}\}} \cong D_2^{\{\bar{c}_k\}}$ if and only if there exists R and $\{c_1, \dots, c_{k-1}\}$ such that

$$T(D_1^{\{\bar{k}\}}) = RT(D_2^{\{\bar{c}_k\}})R', T(D_1^{\{\bar{k}, \bar{k}-1\}}) = RT(D_2^{\{\bar{c}_k, \bar{c}_{k-1}\}})R', \dots, T(D_1^{\{1\}}) = RT(D_2^{\{c_1\}})R'.$$

Then $D_1 \cong D_2$, if and only if $T(D_1) \equiv T(D_2)$ and $D_1^{\{\bar{q}\}} \cong D_2^{\{\bar{c}_q\}}$ for some integers q and c_q .

Lemma 4.2: $\{T(D^{\{\bar{1}\}}), \dots, T(D^{\{\bar{k}\}})\}$ determines $T(D)$.

We show this result for an arbitrary element $T_{ij}(D)$. Suppose we have a design D , with k factors and we know the T matrices for the k projections $\{T(D^{\{\bar{1}\}}), \dots, T(D^{\{\bar{k}\}})\}$

for D . Define $r = \frac{T_{ij}(D) + k}{2}$. Then for r values of $l = 1, 2, \dots, k$, $T_{ij}(D^{(l)}) = T_{ij}(D) - 1$,

and for $k - r$ values of l , $T_{ij}(D^{(l)}) = T_{ij}(D) + 1$. There are two possibilities for $T_{ij}(D)$:

The set $\{T_{ij}(D^{(l)}), \dots, T_{ij}(D^{(k)})\}$ will contain both $T_{ij}(D) - 1$ and $T_{ij}(D) + 1$ values, in which case they bound $T_{ij}(D)$; or the set will contain one constant value, in which case

$T_{ij}(D) = T_{ij}(D^{(l)}) + 1$ if $T_{ij}(D^{(l)})$ is positive, or $T_{ij}(D^{(l)}) - 1$ if $T_{ij}(D^{(l)})$ is negative.

Q.E.D.

Lemma 4.2 states that the set of $\{T(D^{(1)}), \dots, T(D^{(k)})\}$ determines $T(D)$. If we are missing one of the projections from that set, we can still determine $T(D)$.

Corollary 4.1: $k - 1$ members from $\{T(D^{(1)}), \dots, T(D^{(k)})\}$ determine $T(D)$.

The proof is as follows: Suppose we have design D_i , with k factors and we know $k - 1$ of the members from $\{T(D^{(1)}), \dots, T(D^{(k)})\}$. $T_{ij}(D^{(q)})$ will either increase or decrease the value of $T_{ij}(D)$ by one. Recall that $r = \frac{T_{ij}(D) + k}{2}$ and for r values of $l = 1, 2, \dots, k$, $T_{ij}(D^{(l)}) = T_{ij}(D) - 1$, and for $k - r$ values of l , $T_{ij}(D^{(l)}) = T_{ij}(D) + 1$. If we are missing one projection, we can still determine $T_{ij}(D)$. There are two possibilities for $T_{ij}(D)$: The set will contain both $T_{ij}(D) - 1$ and $T_{ij}(D) + 1$ values, in which case they bound $T_{ij}(D)$; or the set will contain one constant value, in which case $T_{ij}(D) = T_{ij}(D^{(l)}) + 1$ if $T_{ij}(D^{(l)})$ is positive, or $T_{ij}(D^{(l)}) - 1$ if $T_{ij}(D^{(l)})$ is negative.

Now we make two conjectures regarding isomorphism of two designs based on isomorphism of their delete-one-factor projections. Let D_1 and D_2 be any regular 2^{k-p} designs with no repeat rows (runs).

Conjecture 4.1: If $D_1^{(i)} \cong D_2^{(c_i)}$ with $i = 1, 2, \dots, k$, where $\{c_1, c_2, \dots, c_k\}$ is any permutation of the integers $\{1, 2, \dots, k\}$, then $D_1 \cong D_2$.

We know under the following conditions that the conjecture is true: Note that

$$T(D_1^{(1)}) + \dots + T(D_1^{(k)}) = (k-1)T(D_1) \text{ and } T(D_2^{(1)}) + \dots + T(D_2^{(k)}) = (k-1)T(D_2).$$

Without loss of generality, assume the columns of D_2 are ordered such that

$$D_1^{(i)} \cong D_2^{(i)} \quad \forall i. \text{ Then there exists an } R_i \ni T(D_1^{(i)}) = R_i T(D_2^{(i)}) R_i'. \text{ If } R_1 = \dots = R_k = R$$

then $T(D_1^{(i)}) = R T(D_2^{(i)}) R' \forall i$ and $\sum T(D_1^{(i)}) = \sum R T(D_2^{(i)}) R'$. Then

$$(k-1)T(D_1) = (k-1)T(D_2). \text{ Thus } T(D_1) = T(D_2) \text{ and } \therefore D_1 \cong D_2.$$

The key requirement of the conjecture is that $\{D_1^{(i)}\} \cong \{D_2^{(i)}\}$ for $i = 1, \dots, k$ implies $T(D_1) \equiv T(D_2)$. We know this requirement is not true in general. In fact, we know that a non-simple design may share the same set of projections as a simple design, but will have a different T matrix. For example consider the 2^4 full factorial design and the replicated 2^{4-1}_{IV} fractional factorial design. While they share the same projections, they have different T matrices.

Define $S \subset \{1, 2, \dots, k\}$ with cardinality s . If Conjecture 4.1 is true, then we suppose that the following stronger conjecture may also be true.

Conjecture 4.2 If two designs D_1 and D_2 , have s projections in common, and these s projections of D_1 , $\{D_1^{(i)} : i \in S\}$ determine $T(D_1)$, then $D_1 \cong D_2$.

Assume we have two designs, D_1 and D_2 , with s projections in common,
 $D_1^{(\bar{i})} \cong D_2^{(\bar{i})}$ for $i \in S$. If the s projections of D_1 , $\{D_1^{(\bar{i})} : i \in S\}$ determine $T(D_1)$, then
they also determine $T(D_2)$ and we suppose $D_1 \cong D_2$.

5. Advantages and Uses of the T Matrix

Hedayat, Sloane, and Stufken's definition 3.4 (1999) states that an orthogonal array $OA(N, k, 2, t)$ with levels from $GF(2)$ is said to be linear if it is simple (runs are distinct) and if, when considered as k -tuples from $GF(2)$, its N runs form a vector space over $GF(2)$ (i.e., satisfy the condition that if R_1 and R_2 are any two runs of the array then every k -tuple $c_1R_1 + c_2R_2$ is also a run, for any choice of $c_1, c_2 \in GF(2)$).

It is known that all two-level regular fractional factorial designs are $OA(N, k, 2, t)$ with $t = (\text{resolution} - 1)$. All regular fractional factorial designs without repeat runs are simple. Fractional factorial designs with a defining relation (regular design) are a subclass of orthogonal arrays and are linear codes (Hedayat, Sloane, and Stufken p.276). Therefore we can take the sum of any two rows from a regular fractional factorial design and using modulus(2) arithmetic it will equal another row in the design. Note that the element-wise product for two runs with levels ± 1 is equivalent to modulus(2) arithmetic for the same two runs with levels 0 and 1. Hence, for regular two level fractional factorial design with levels of ± 1 , any two rows multiplied element-wise will result in another row of the design.

For example consider a 2_{III}^{5-2} regular fractional factorial design where:

$$D = \begin{matrix} & -1 & -1 & -1 & -1 & 1 \\ & -1 & -1 & 1 & 1 & -1 \\ & -1 & 1 & -1 & 1 & 1 \\ D = & -1 & 1 & 1 & -1 & -1 \\ & 1 & -1 & -1 & 1 & -1 \\ & 1 & -1 & 1 & -1 & 1 \\ & 1 & 1 & -1 & -1 & -1 \\ & 1 & 1 & 1 & 1 & 1 \end{matrix}$$

and the T matrix is:

$$T(D) = DD' = \begin{bmatrix} 5 & -1 & 1 & -1 & -1 & 1 & -1 & -3 \\ -1 & 5 & -1 & 1 & 1 & -1 & -3 & -1 \\ 1 & -1 & 5 & -1 & -1 & -3 & -1 & 1 \\ -1 & 1 & -1 & 5 & -3 & -1 & 1 & -1 \\ -1 & 1 & -1 & -3 & 5 & -1 & 1 & -1 \\ 1 & -1 & -3 & -1 & -1 & 5 & -1 & 1 \\ -1 & -3 & -1 & 1 & 1 & -1 & 5 & -1 \\ -3 & -1 & 1 & -1 & -1 & 1 & -1 & 5 \end{bmatrix}.$$

Note that each column (and row) of T have the same distribution of values. For instance, each column contains the values -3, -1, 1, and 5 with frequencies 1, 4, 2, and 1, respectively.

Theorem 5.1: Any two-level regular factorial design D will have a constant column distribution in $T(D)$.

We now show that the elements of t_i^D are a permutation of the elements of t_j^D for arbitrary i and j from $\{1, \dots, n\}$. We know that $x_i x_j = x_l$ for some $l \in \{1, 2, \dots, n\}$, where $x_i x_j$ is defined as the element-wise product of the i^{th} and j^{th} rows. Hence, $x_i x_l = x_j$.

Now define $t_j^D = D \cdot x_j$ where x_j' is the j^{th} row of D , and rewrite $t_j^D = \begin{bmatrix} x_1' x_j \\ \vdots \\ x_n' x_j \end{bmatrix}$ using the

specified i^{th} and j^{th} rows above as $t_j^D = \begin{bmatrix} x_1' (x_i x_l) \\ \vdots \\ x_n' (x_i x_l) \end{bmatrix} = \begin{bmatrix} (x_i x_l)' x_1 \\ \vdots \\ (x_i x_l)' x_n \end{bmatrix}$. From the definition of

a group we know that any element from a group multiplied by the group results in the

original group. Therefore this implies that the matrix $= \begin{bmatrix} (x_1 x_l)' x_i \\ \vdots \\ (x_n x_l)' x_i \end{bmatrix}$ contains all the

elements of t_i^D . Q.E.D.

6. Functions of the T Matrix

We know from Theorem 5.1 that t_1^D, \dots, t_n^D are simply different permutations of the same vector. Butler (2003) states that $T_{ij}(D)$ measures the confounding between the i^{th} and j^{th} rows. He defines $\mu_k = n^{-2} \sum_{i=1}^n \sum_{j=1}^n T_{ij}^k(D)$ as the k^{th} moment of the elements of the T matrix. Therefore, the moments μ_0, \dots, μ_k provide an overall measure of the confounding between rows of the design (Butler 2003). By Theorem 5.1 we can use any one column of the T matrix to calculate the moments of a regular design. When our use of t_i^D does not depend on the subscript i , we simply write t^D to represent an arbitrary column of T . We know from Butler (2003) that the design moments for D can be used to compare and rank designs. The design moments method results in an identical ranking of designs that results from using the word length pattern for designs (Butler 2003). Since the word length pattern and moments of T are both functions of t^D , it is possible that t^D might be more discriminating than the moments of a design or equivalently the word length pattern. However; by Theorem 6.1, the frequencies of t^D can be written as a function of the moments, so t^D is no more discriminating than is the word length pattern.

Let f_0, \dots, f_k represent the frequency of values for $-k, (-k+2), \dots, k$, respectively, in t^D .

Theorem 6.1: The frequencies f_0, \dots, f_k are a function of the moments μ_0, \dots, μ_k .

$$\text{We can write } n\mu_j = \sum_{i=0}^k (2i-k)^j f_i \text{ for } j \in \{0, 1, \dots, k\}$$

Note that: $n\mu_0 = \sum_{i=0}^k (2i-k)^0 f_i = \sum_{i=0}^k f_i = n$. Define $\mu_j' = \sum_{i=0}^k i^j f_i / n$ and let

$$M = \begin{bmatrix} \mu_0 \\ \vdots \\ \mu_k \end{bmatrix} \text{ and } M^* = \begin{bmatrix} \mu_0' \\ \vdots \\ \mu_k' \end{bmatrix}. \text{ Note that } M = BM^* \text{ where } B \text{ is a lower triangular matrix}$$

with positive values on the diagonal since $\mu_r = E[2i-k]^r = 2^r E[i^r] - 2^{r-1}rkE[i^{r-1}] + \dots = 2^r \mu_r' - 2^{r-1}rk\mu_{r-1}' + \dots$. We know that the determinant of a triangular matrix is equal to the product of the elements along the diagonal (Eves, p123). Hence, $M^* = B^{-1}M$ since the matrix B is nonsingular and can be inverted.

Now write the moments of a design, μ_0', \dots, μ_k' , as a system of equations

$$nM^* = AF \text{ where } F = \begin{bmatrix} f_0 \\ \vdots \\ f_k \end{bmatrix}_{(k+1) \times 1} \text{ and the coefficient matrix } A \text{ is:}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 2 & 3 & \dots & k \\ 0 & 1 & 2^2 & 3^2 & \dots & k^2 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 1 & 2^k & 3^k & \dots & k^k \end{bmatrix}_{(k+1) \times (k+1)}$$

The determinant of matrix A can be described as a Vandermonde determinant (Eves p.127). From this literature, it is known that A is nonsingular (since the values of A are integer and increasing $[0, 1, \dots, k]$). Thus A can be inverted, so we can rewrite our system of equations in terms of $F = A^{-1}nM^*$ and $F = nA^{-1}B^{-1}M$. This means the frequencies, F , are a function of the moments M . Therefore the probabilities that generate those

moments are unique and the moments are unique in the sense that any two designs with the same moments M must have identical t^D frequencies F .

Since word length pattern, or equivalently t^D , is unsuccessful in distinguishing many designs at $n = 64$ and larger, we are interested in creating a more discriminating function from pairs of columns of T . Let $T2^D$ represent the set of n pairs of columns of T for D where $T2^D = \{(t_1^D, t_1^D), (t_1^D, t_2^D), \dots, (t_1^D, t_n^D)\}$.

Define $G(T2^D) = \{g(t_1^D, t_1^D), g(t_1^D, t_2^D), \dots, g(t_1^D, t_n^D)\}$, where $g(t_1^D, t_v^D) = \sum_{r=1}^n h(T_{r1}T_{rv})$

and $h(x) = 0$ when $x \leq 0$, and $h(x) = x^{-1}$ when $x > 0$. For example, consider the 2_{III}^{5-2} regular design again. The t^D vector contains the values -3, -1, 1, and 5, with frequencies 1, 4, 2, and 1, respectively. Figure 6.1 shows the four bivariate frequency distributions that occur for the pairs of columns for T . While the columns of T have identical frequency distributions, the pairs of columns for T do not. For the n pairs (t_1^D, t_j^D) $j = 1, 2, \dots, n$, four possibilities occur with frequencies 1, 4, 2, and 1, respectively (see Figure 6.1). Therefore, $G(T2^D) = \{6.1511, 0.667, 4.4, 0.667, 0.667, 4.4, 0.667, 6.0\}$ for this design. We sort this set for our convenience in comparing designs so that $G(T2^D) = \{0.667, 0.667, 0.667, 0.667, 4.4, 4.4, 6.0, 6.1511\}$.

We chose to define $T2^D$ above pairing each of the n columns of T with t_1^D . We now show that the set $G(T2^D)$ is invariant to the choice of which column we fix.

Lemma 6.1: For any $i \in (1, \dots, n)$, $\{g(t_i^D, t_j^D) \mid j = 1, \dots, n\} = \{g(t_n^D, t_{r_j}^D) \mid j = 1, \dots, n\}$ where (r_1, \dots, r_n) is a permutation of $(1, \dots, n)$.

(1, j) Pairs of T matrix columns:		Bivariate distribution:					$g(t_1^D, t_j^D)$
(1, 1)		-3	-1	1	5	totals	
	-3	1				1	
	-1		4			4	= 6.1511
	1			2		2	
	5				1	1	
	totals	1	4	2	1	8	
(1, 2)		-3	-1	1	5	totals	
	-3		1			1	
	-1	1		2	1	4	= 0.667
	1		2			2	
	5		1			1	
	totals	1	4	2	1	8	
(1, 3)		-3	-1	1	5	totals	
	-3			1		1	
	-1		4			4	= 4.4
	1	1			1	2	
	5			1		1	
	totals	1	4	2	1	8	
(1, 8)		-3	-1	1	5	totals	
	-3				1	1	
	-1		4			4	= 6.0
	1			2		2	
	5	1				1	
	totals	1	4	2	1	8	

Figure 6.1: 2_{III}^{5-2} T Matrix, Pairs of Columns

Without loss of generality, assume x_n is the treatment combination with all +1 levels. Then the element-wise product of $x_i x_j = x_n x_{r_j} = x_{r_j}$ for some $r_j \in \{1, \dots, n\}$. Hence, Lemma 6.1. By this lemma, $\{g(t_i^D, t_j^D) \mid j = 1, \dots, n\}$ is invariant to the choice of i . We defined $T2^D$ with $i = 1$.

Theorem 6.2: $D_1 \cong D_2 \Rightarrow G(T2^{D_1}) = G(T2^{D_2})$

Since $D_1 \cong D_2$, there exists $\{r_1, \dots, r_n\}$, a permutation of $\{1, \dots, n\}$, such that $T(D_1) = RT(D_2)R'$ where R is the permutation matrix defined as $R_{ij} = 1$ if $j = r_i$, and zero otherwise. Then $(t_1^{D_1}, t_j^{D_1}) = (Rt_{r_1}^{D_2}, Rt_{r_j}^{D_2})$ for $j = (1, \dots, n)$. So $g(t_1^{D_1}, t_j^{D_1}) = g(t_{r_1}^{D_2}, t_{r_j}^{D_2})$ for $j = (1, \dots, n)$, because the permutation matrix R does not affect the computation of $g(\cdot, \cdot)$ since we are summing the rows. Then by Lemma 6.1, $\{g(t_{r_1}^{D_2}, t_{r_j}^{D_2}) \mid j = 1, \dots, n\} = G(T2^{D_2})$ and so $G(T2^{D_1}) = G(T2^{D_2})$. Q.E.D.

The set $G(T2^D)$ uniquely identifies all regular resolution IV designs for $n < 128$. At $n = 128$, $G(T2^D)$ uniquely identifies 296,958 of the 296,960 even/odd designs (it does not uniquely identify 2 even/odd designs) which differ based on their delete-one-factor projections. However, it does distinguish the two 2_{VII}^{31-16} regular designs that are commonly cited from Chen and Lin (1991) as an example of non-isomorphic designs with common letter pattern matrices. See Section 11 for more comparisons with other common criterion.

7. Exhaustive Even/Odd Design Search Method

We now present a new method for finding minimum aberration designs using a build up and delete-one-factor projection strategy. As noted previously, CSW were unable to fully enumerate designs beyond $k = 11$ at $n = 128$, due to the enormous computations required to perform their isomorphism checks. Our approach for regular factorial designs attempts to take advantage of a simplified isomorphism check. Using Conjecture 4.1 we replace the permutation check for isomorphism from Clark and Dean and check the set of delete-one-factor projections for each design. We save only the unique sets of delete-one-factor projections and the $G(T2^D)$ set, thus determining our non-isomorphic designs.

If Conjecture 4.1 is not true, then there could exist designs with non-equivalent T matrices that have a common set of delete-one-factor projections. We differentiated designs based on their delete-one-factor projections. We did not check $G(T2^D)$ simultaneously with the delete-one-factor projections and therefore did not have the occasion to find any designs with isomorphic delete-one-factor projections but different sets of $G(T2^D)$, which would provide a counter-example to Conjecture 4.1 for $n = 128$.

The approach is as follows: begin with all non-isomorphic resolution IV designs with k factors. Consider all possible $k + 1$ factor designs obtained by adding a generator to each k factor design. We then check the $k + 1$ delete-one-factor projections. If the $k + 1$ delete-one-factor projections for design D_1 are equal to the $k + 1$ one-factor projections for D_2 then the designs are considered isomorphic by Conjecture 4.1; otherwise they are non-isomorphic. This process can be repeated as we increase k by one factor at a time.

Using this approach allowed us to complete an "exhaustive" search of even/odd designs at $n = 128$ for $k \leq 40$.

Another step to reduce the computational burden at $n = 128$ was the elimination of the requirement to retain even designs past $k = 22$. This was possible for the following reasons. Resolution IV 2^{k-p} even/odd designs project to a set of $k - m$ $2^{(k-1)-(p-1)}$ even/odd designs and m isomorphic $2^{(k-1)-(p-1)}$ even designs (by Lemma 2.1), where m is defined as the multiplicity for the number of delete-one-factor projections from a 2^{k-p} design that project to a $2^{(k-1)-(p-1)}$ even design.

We classify m into three cases: When $m = 0$, the set of k projections are all $2^{(k-1)-(p-1)}$ even/odd designs and by Lemma 4.2 we can determine $T(D)$. When $m = 1$, we use Conjecture 4.2, motivated by Corollary 4.1 and the set of $k - 1$ even/odd $2^{(k-1)-(p-1)}$ designs to determine D . The last case, when $m > 1$, is determined as follows: We know $k - m$ projections are $2^{(k-1)-(p-1)}$ even/odd designs and m projections are isomorphic $2^{(k-1)-(p-1)}$ even designs. Without loss of generality, suppose $D^{(i)}$ $i = 1, \dots, m$ are $2^{(k-1)-(p-1)}$ even designs, and the remaining $k - m$ projections are $2^{(k-1)-(p-1)}$ even/odd designs. Then $G(T2^{D^{(i)}})$, m , and $D^{(i)}$ ($i > m$) determine D (up to isomorphism). The reason is as follows: for $n = 8, 16, 32$, and 64 , we know that $G(T2^D)$ uniquely distinguishes all 2_{IV}^{k-p} designs. For $n = 128$, even $2_{IV}^{(k-1)-(p-1)}$ designs projected from 2_{IV}^{k-p} designs with $m \geq 2$, permit us to distinguish D by $G(T2^D)$ since the even $2_{IV}^{(k-1)-(p-1)}$ designs can be written as the product array $2^1 \times 2^{(k-2)-(p-1)}$ and so all are uniquely distinguished by $G(T2^D)$.

8. Resolution IV Designs of Size 128

We characterize the even/odd resolution IV design for $n = 128$ using five criterion:

- wlp (minimum aberration)
- Maximum degrees of freedom used for main effects and two-factor interactions
- Minimum L_{\max} (the length of the longest two-factor interaction alias chain)
- Maximum number of clear two-factor interactions
- Minimum CD2 (the unique portion of the centered L2 discrepancy from Ma, Fang, and Lin 2001).

The minimum aberration designs for $k \leq 40$ at $n = 128$ are listed in Table 8.1 along with the above criteria and their respective ranking. The complete alp is also provided for each design. Appendix C contains a catalog of the best even/odd designs and their rankings for $k = 8, \dots, 40$ with respect to our various criteria.

Our exhaustive search of even/odd designs found not only the minimum aberration designs, but also a number of interesting results. All minimum aberration designs from $10 \leq k \leq 40$ are even/odd designs. We found that the uniform centered design criteria (Ma, Fang, and Lin 2001) is closely related to the word length pattern. Our calculation of the minimum CD2* value agreed with the minimum aberration design in all but four cases; in those cases, the minimum aberration value was the second smallest CD2* value.

Table 8.1: Minimum Aberration Regular Resolution IV (or higher) Designs for $n = 128$

Table 8.1: Minimum Adaptation Regular Resolution IV (or higher) Designs for $n = 126$																						
Design	w ₄	w ₅	w ₆	w ₇	w ₈	alp	df	C2FI	Lmax	df	C2FI	Lmax	rank	CD2*	rank	CD2*	rank					
8-1.1	0	0	0	0	1	28	36	28	1	1	1	1	1	55.09	1	55.09	1					
9-2.1	0	0	3	36			45	36	1	1	1	1	1	49.59	1	49.59	1					
10-3.1	0	3	3	45			55	45	1	1	1	1	1	44.63	1	44.63	1					
11-4.1	0	6	6	55			66	55	1	1	1	1	1	40.17	1	40.17	1					
12-5.1	1	8	12	60	3		75	60	2	1	1	1	1	36.16	1	36.16	1					
13-6.1	2	16	18	66	6		85	66	2	1	1	1	1	32.55	1	32.55	1					
14-7.1	3	24	36	73	9		96	73	2	1	1	1	1	29.30	1	29.30	1					
15-8.1	7	32	52	63	21		99	63	2	2	11	1	1	26.39	1	26.39	1					
16-9.1	10	48	72	60	30		106	60	2	2	24	1	1	23.77	1	23.77	1					
17-10.1	15	60	130	46	45		108	46	2	53	1594	1	1	21.42	1	21.42	1					
18-11.1	20	80	200	33	60		111	33	2	209	10601	1	1	19.30	1	19.30	1					
19-12.1	27	120	235	36	54	9	118	36	3	22	5807	1	1	17.40	1	17.40	1					
20-13.1	36	152	340	24	60	14	119	24	4	111	28084	1	1	15.69	1	15.69	1					
21-14.1	51	200	414	26	54	15	123	26	5	23	17819	45	1	14.17	2	14.17	2					
22-15.1	65	248	572	25	36	32	124	25	6	20	14585	942	1	12.80	1	12.80	1					
23-16.1	83	316	744	12	52	24	125	12	7	10	32307	5495	1	11.57	1	11.57	1					
24-17.1	102	384	992	0	54	16	122	0	6	120	27865	4	1	10.46	1	10.46	1					
25-18.1	124	482	1312	0	64	0	127	0	5	1	20240	1	1	9.469	1	9.469	1					
26-19.1	152	568	1704	0	29	41	124	0	6	13	13068	1	1	8.579	1	8.579	1					
27-20.1	180	690	2200	0	15	55	125	0	6	6	7696	1	1	7.779	1	7.779	1					
28-21.1	210	840	2800	0	0	70	126	0	6	2	3930	1	1	7.061	1	7.061	1					
29-22.1	266	945	3472	0	0	70	127	0	7	1	1914	1	1	6.431	1	6.431	1					
30-23.1	335	972	4662	0	0	40	117	0	11	773	799	182	1	5.866	1	5.866	1					
31-24.1	391	1134	5826	0	0	24	118	0	12	323	331	96	1	5.352	1	5.352	1					
32-25.1	452	1322	7219	0	0	12	119	0	13	130	125	46	1	4.891	2	4.891	2					
33-26.1	518	1543	8863	0	0	4	120	0	14	67	67	27	1	4.478	2	4.478	2					
34-27.1	589	1800	10788	0	0	0	121	-	15	11	-	1	1	4.108	2	4.108	2					
35-28.1	665	2100	13020	0	0	0	122	-	15	3	-	1	1	3.776	1	3.776	1					
36-29.1	756	2401	15736	0	0	0	123	-	16	2	-	1	1	3.481	1	3.481	1					
37-30.1	854	2744	18886	0	0	0	124	-	17	1	-	1	1	3.216	1	3.216	1					
38-31.1	959	3136	22512	0	0	0	125	-	18	1	-	1	1	2.979	1	2.979	1					
39-32.1	1071	3584	26656	0	0	0	126	-	19	1	-	1	1	2.767	1	2.767	1					
40-33.1	1190	4096	31360	0	0	0	127	-	20	1	-	1	1	2.576	1	2.576	1					
alp – alias length pattern of two-factor interactions							df – total degrees of freedom used for main effects and two-factor interactions							CD2* – unique portion of uniformity measure value from Ma, Fang, Lin (2001)								
wlp – word length pattern							alp – alias length pattern of two-factor interactions															
C2FI – clear two-factor interactions							Lmax – length of longest alp chain															

df – total degrees of freedom used for main effects and two-factor interactions
CD2* – unique portion of uniformity measure value from Ma, Fang, Lin (2001)

wlp – word length pattern
C2FI – clear two-factor interactions
alp – alias length pattern of two-factor interactions
Lmax – length of longest alp chain

No minimum aberration designs have any clear two-factor interactions beyond $k = 23$, although we found designs with clear two-factor interactions up to $k = 33$. We know from Chen and Hedayat (1998) that designs with clear two-factor interactions exist only if $k \leq n/4 + 1$. In general, as the number of factors increases, the number of good designs (based on word length pattern) with clear two-factor interactions decreases.

There exist 296,960 even/odd non-isomorphic resolution IV (or higher) designs for $n = 128$ (see Table 8.2). There are also 88 resolution IV sos designs, and all but one of the sos designs are even/odd designs. We also now know that sos designs may have the same word length patterns but different alp and may even share the same word length pattern as other non-sos designs. For instance, consider the three designs at $k = 33$, where the two sos designs 33-26.42b and 33-26.42c share identical word length patterns with design 33-26.42a which is not an sos design. All three designs have different alias length patterns.

We also found two notably good sos designs: $k = 29$, and $k = 40$. The design at $k = 40$ is well known and many of its projections lead to other minimum aberration designs. The sos design at $k = 29$ has a remarkably smaller number of length-four words than any other $k = 29$ design and several of the sos design's projections are also minimum aberration designs. In particular, the minimum aberration designs can be found by projecting from sos designs at $k = 29$ or $k = 40$ for $k = 40, 39, \dots, 26, 24, 16, 13, 11, 10$, and 9 (see Section 12).

It is interesting to note that for $k \leq 40$, the minimum aberration design word length pattern for each k is indeed unique, which supports the conjecture that the word length pattern is unique for minimum aberration resolution IV designs. In fact, only

Table 8.2: Existence of Resolution IV⁺ designs

k	# of even/odd designs, $n = 64$	# of even designs, $n = 64$	# of even/odd designs, $n = 128$	# of even designs, $n = 128$
7	2	2	-	-
8	3	4	2	3
9	6	6	7	6
10	12	12	19	14
11	20	14	62	30
12	22	21	180	69
13	24	23	487	136
14	20	29	1,240	295
15	15	29	2,926	596
16	11	37	6,208	1,292
17	10	30	11,787	2,651
18	3	30	19,466	5,598
19	1	24	27,994	11,341
20	1	23	35,192	22,728
21	-	16	39,201	43,516
22	-	15	38,847	79,603
23	-	9	34,868	?
24	-	8	28,133	?
25	-	5	20,569	?
26	-	4	13,498	?
27	-	2	8,075	?
28	-	2	4,284	?
29	-	1	2,149	?
30	-	1	976	?
31	-	1	433	?
32	-	1	197	?
33	-	-	101	?
34	-	-	31	?
35	-	-	13	?
36	-	-	8	?
37	-	-	3	?
38	-	-	2	?
39	-	-	1	?
40	-	-	1	?

at $k = 31$, does one have to go beyond length-5 words in the defining relation to differentiate minimum aberration designs from weak minimum aberration designs.

Finally, the L_{\max} results show that it is impossible to create an $n = 384$ $3/4$ -design (John 1962) for $k \geq 20$ from resolution IV fractions, since $L_{\max} > 3$. Also many of the better designs based on word length pattern are also ranked in the best designs according to L_{\max} . For example, the top eight designs based upon word length pattern are also the top eight ranked designs for L_{\max} at $k = 18$.

9. Incomplete Enumeration of Designs Based on Word Length Pattern

As the size of n increases, more and more computer resources are required to fully enumerate designs. The next two sections explore computationally simpler (imperfect) isomorphism checks in order to evaluate their potential merit for $n = 256$ and beyond.

Butler developed an algorithm using a flawed isomorphic rule based on the moments of the designs (word length pattern) that starts with a basic set of factors and then adds one generator at a time to construct new designs. He describes his approach as follows:

"The iterative algorithm uses all the designs with distinct wordlength patterns (or equivalently, distinct T moments) for k factors and adds an extra factor to each to form designs for $k + 1$ factors. Only designs with distinct wordlength patterns are retained for the next stage of the algorithm. At each stage, the wordlength pattern is determined from the elements of T . The algorithm does not recognize that on rare occasions designs with the same wordlength pattern are not necessarily isomorphic. However, a design for k factors can be formed from any of the k projections involving $k - 1$ factors and so designs are highly unlikely to be lost altogether." (Butler 2002b)

Using Butler's methodology, we were able to easily search for even/odd resolution IV designs using Matlab version 6.5 on a Pentium III and IV computer.

Our program constructed a full factorial in seven basic factors for $n = 128$ runs and then constructed a generator matrix of all possible generators (based on the 120 different interactions involving the basic columns). We then started with the seven basic factors and added one generator at a time. We calculated t^D for each design and retained only one design for each distinct t^D vector. This method does not distinguish between non-isomorphic designs with identical design moments (word length patterns). In our implementation, this method was successful in finding all minimum aberration designs except at $k = 24$, where we found only the weak minimum aberration design. In general,

we lost about two percent of the word length patterns using this approach at $n = 128$ runs (see Table 10.1). However, we only identified 20% of the even/odd designs that exist. Thus having non-isomorphic designs with the same wlp is a very common occurrence at $n = 128$. For example, the word length pattern $(0, 0, 0, 8, 34, 42, \dots)$ at $k = 15$, occurs for four designs (see p. 106). Another word length pattern $(0, 0, 0, 21, 0, 80, \dots)$ at $k = 15$, occurs for 48 non-isomorphic designs.

10. An Improved Imperfect Isomorphic Rule Approach

In an effort to find a more discriminating function than t^D (or equivalently, wlp) for our imperfect isomorphic rule approach to determining isomorphic designs, we turned to the $G(T2^D)$ vector. $G(T2^D)$ uniquely determined the same designs cataloged by Sun (2001) and CSW (1993) for $n = 128$ and $k = 8, 9, 10, 11$ as well as all designs at $n = 64$. Although we know that several non-isomorphic designs do have identical $G(T2^D)$ sets, this happened in only rare instances (see Table 10.1). This means that only those designs with unique $G(T2^D)$ vectors are kept as we sequentially build up our designs. While this method does miss some designs, the $G(T2^D)$ vector is much more discriminating than t^D .

The empirical results at $n = 128$ show that the designs that were lost were not the better designs in terms of word length pattern, and that although a few (57) non-isomorphic designs were missed, other designs with identical word length pattern, alias length pattern, and number of clear two-factor interaction effects were found.

Table 10.1 lists the number of even/odd designs found using several different isomorphic checks for $n = 128$ and $k \leq 40$. We show the number of even/odd designs found using the word length pattern as a simple but flawed isomorphic rule, and the number of even/odd designs found using $G(T2^D)$ as a flawed isomorphic rule. We also show the complete enumeration of all even/odd designs and the number of unique word length patterns that exist among the exhaustive list obtained based on delete-one-factor projections. We also provide percentages of designs found using the different

Table 10.1: Comparison of Methods for Finding Even/Odd Resolution IV⁺ Designs

k	# of e/o designs by projections	# of unique e/o wlp by projections	t^D # of e/o designs found	% found of e/o unique wlp	% found of total e/o designs	$G(T2^D)$, # of e/o designs found	% found of total e/o designs
8	2	2	2	100	100	2	100
9	7	7	7	100	100	7	100
10	19	18	18	100	94.7	19	100
11	62	48	48	100	77.4	62	100
12	180	118	118	100	65.6	180	100
13	487	243	243	100	49.9	487	100
14	1,240	448	444	99.1	35.8	1,240	100
15	2,926	777	765	98.5	26.1	2,925	99.9
16	6,208	1,278	1,257	98.4	20.2	6,208	100
17	11,787	1,996	1,946	97.5	16.5	11,787	100
18	19,466	2,890	2,825	97.8	14.5	19,466	100
19	27,994	4,051	3,937	97.2	14.1	27,993	99.9
20	35,192	5,211	5,109	98	14.5	35,192	100
21	39,201	6,237	6,086	97.6	15.5	39,201	100
22	38,847	6,546	6,422	98.1	16.5	38,847	100
23	34,868	6,361	6,226	97.9	17.8	34,868	100
24	28,133	5,656	5,578	98.6	19.8	28,133	100
25	20,569	4,709	4,629	98.3	22.5	20,569	100
26	13,498	3,575	3,516	98.4	26.0	13,498	100
27	8,075	2,611	2,547	97.5	31.5	8,075	100
28	4,284	1,720	1,691	98.3	39.5	4,284	100
29	2,149	1,119	1,099	98.2	51.1	2,149	100
30	976	632	620	98.1	63.5	976	100
31	433	340	332	97.6	76.7	433	100
32	197	177	175	98.9	88.8	197	100
33	101	90	90	100	89.1	101	100
34	31	30	30	100	96.8	31	100
35	13	13	13	100	100	13	100
36	8	8	8	100	100	8	100
37	3	3	3	100	100	3	100
38	2	2	2	100	100	2	100
39	1	1	1	100	100	1	100
40	1	1	1	100	100	1	100

approaches. In no cases did the sets of delete-one-factor projections fail to distinguish designs with different t^D or $G(T2^D)$.

11. Interesting Designs of Size 128

While letter pattern and $G(T2^D)$ are more discriminating than wlp, neither is universally more successful. For example, at $k = 11$ we found non-isomorphic designs with distinct $G(T2^D)$ values and identical letter pattern matrices, while at $k = 15$ we found non-isomorphic designs with identical $G(T2^D)$ (and identical bivariate distributions) but distinct letter pattern matrices.

During the exhaustive search for designs, a number of interesting designs were encountered in trying to determine non-isomorphic designs. We describe four problem cases of interest. Below is a sample of some of the designs encountered along with a short description of the designs and their properties.

Problem Case 1:

The first case occurs at $k = 11$. Let pc11a, pc11b, and pc11c represent the three problem designs. All three even/odd designs have the same word length pattern and the same alias length pattern. The first design, pc11a, has a different letter pattern matrix than pc11b and pc11c. The other two designs, pc11b and pc11c, have identical letter pattern matrices. All three designs have unique $G(T2^D)$ values. Table 11.1 lists the generators for these designs.

Table 11.1: $k = 11, n = 128$ Problem Designs

Design	Generators
pc11a	7 25 43 116
pc11b	7 45 56 91
pc11c	7 56 77 91

Problem Case 2:

The second case occurs at $k = 15$. These even/odd designs have identical $G(T2^D)$ values, identical word length patterns, and identical alias length patterns. However, the letter pattern matrix for each design is different. Table 11.2 lists the generators for these designs.

Problem Case 3:

The third case occurs at $k = 16$. There are 18 pairs of designs that have various $G(T2^D)$ values. Each pair of designs also have identical word length patterns and identical letter pattern matrices respectively. The designs do have different alias length patterns. The first four designs listed below are even/odd designs (a1 through b2) and the remaining designs are even. Table 11.3 lists the generators for these designs.

Table 11.2: $k = 15, n = 128$ Problem Designs

Design	Generators
pc15a	7 11 19 38 59 73 100 120
pc15b	7 11 19 38 62 73 97 120

Table 11.3: $k = 16, n = 128$ Problem Designs

Design	Generators								
pc16a1	7	11	19	41	52	61	74	101	120
pc16a2	7	11	19	35	61	62	73	85	120
pc16b1	7	11	21	38	57	73	82	93	120
pc16b2	7	11	19	38	57	73	84	93	120
pc16c1	7	11	21	26	31	112	121	122	124
pc16c2	7	11	21	25	31	112	121	122	124
pc16d1	7	25	42	55	79	112	121	122	124
pc16d2	7	25	31	42	52	112	121	122	124
pc16e1	7	11	21	26	52	84	121	122	124
pc16e2	7	25	26	47	79	112	121	122	124
pc16f1	7	13	21	104	110	112	118	121	122
pc16f2	7	11	13	19	100	103	121	122	124
pc16g1	7	13	28	35	62	104	112	121	122
pc16g2	7	19	28	41	79	112	121	122	124
pc16h1	7	13	28	38	59	104	112	121	122
pc16h2	7	19	31	41	79	112	121	122	124
pc16i1	7	13	38	61	91	104	112	121	122
pc16i2	7	13	22	38	59	104	112	121	122
pc16j1	7	13	44	55	104	110	112	121	122
pc16j2	7	13	38	59	61	104	112	121	122
pc16k1	7	13	44	79	104	110	112	121	122
pc16k2	7	13	38	59	91	104	112	121	122
pc16l1	7	38	61	69	94	104	112	121	122
pc16l2	7	13	22	59	91	104	112	121	122
pc16m1	7	13	22	44	49	62	112	121	122
pc16m2	7	13	44	59	91	104	112	121	122
pc16n1	7	13	22	44	49	82	112	121	122
pc16n2	7	13	44	55	59	104	112	121	122
pc16o1	7	19	28	35	61	76	112	121	122
pc16o2	7	28	38	47	61	104	112	121	122
pc16p1	7	21	25	47	55	84	112	121	122
pc16p2	7	28	38	47	59	104	112	121	122
pc16q1	7	11	19	38	44	52	100	121	122
pc16q2	7	13	21	38	59	104	112	121	122
pc16r1	7	11	19	38	44	100	103	121	122
pc16r2	7	13	21	59	91	104	112	121	122

Problem Case 4:

The fourth case occurs at $k = 19$. The following two pairs of designs have identical $G(T2^D)$ values, word length pattern, alias length pattern, and letter pattern matrices respectively. They are only distinguished by their sets of delete-one-factor projections. The first pair (pc19a1 and pc19a2) are even designs, the second pair are even/odd designs. Table 11.4 lists the generators for these designs.

Table 11.4: $k = 19, n = 128$ Problem Designs

Design	Generators											
pc19a1	7	13	22	44	49	62	91	98	112	118	121	122
pc19a2	7	13	22	44	49	62	91	98	112	121	122	124
pc19b1	7	11	25	31	35	50	85	104	112	121	122	124
pc19b2	7	11	25	31	35	50	86	104	112	121	122	124

12. Finding Good Designs Using Naïve Projections

As noted previously, the difficulty of finding minimum aberration designs (and other good designs) increases as n becomes larger. Examining the case of $n = 64$ suggests that sequentially eliminating factors to minimize the number of length four words in the resulting design (ties broken by the minimization of length-five words, then length-six words, etc.) from a relatively few sos designs present a few design arrays from which good (minimum aberration) designs are found. This method will be referred to as the naïve projection approach.

Table 12.1 lists the number of length-four words (w_4) for minimum aberration designs and for the naïve projections from each of the eight even/odd sos designs for $n = 64$. The naïve projections that result in the minimum aberration design are marked with "*", while those projections resulting in a weak minimum aberration design are marked with "**".

Table 12.1: Number of Length-Four Words for SOS Naïve Projections, $n = 64$

k	MA	sos20	sos18	sos17b	sos17d	sos17e	sos17g	sos17j	sos13
20	125	125*							
19	100	100*							
18	78	78*	92						
17	59	59*	68	60	65	68	73	105	
16	43	43*	49	45	45	49	53	77	
15	30	30*	34	33	33	33	37	55	
14	22	22*	22**	23	23	23	24	38	
13	14	15	14*	15	15	15	16	25	14**
12	6	9	8	10	10	10	10	15	6*
11	4	5	4*	6	6	6	5	9	4*
10	2	2*	2*	3	3	3	3	5	2*
9	1	1*	1*	1*	1*	1*	1*	2	1*
8	0,2	0*	0*	0*	0*	0*	0*	0*	0*

* = minimum aberration; ** = weak minimum aberration

It is interesting to note that the 20-factor sos design projects to the minimum aberration design for $k = 14, 15, \dots, 20$ (and also 8, 9, and 10); the 13-factor sos design is weak minimum aberration at $k = 13$, and its naïve projections are minimum aberration for $k = 8, 9, \dots, 12$. The weak minimum aberration sos design at $k = 13$ has 36 clear two-factor interactions, 16 more than the minimum aberration design and is arguably preferred over the minimum aberration design due to the more clear two-factor interactions.

Since sequential projection from just two $n = 64$ run designs provide attractive designs for all $k = 8, 9, \dots, 20$, we list these two sos designs in Table 12.2, arranging the design columns so that one only needs to include the number of generators that correspond to the desired number of factors. For instance, for the minimum aberration 18-factor design, simply omit the last two columns of the 20-factor design. The 20-14.a sos design is recommended for $k = 14, \dots, 20$ and the 13-7.b design for $k = 8, \dots, 13$. These designs and their embedded projections are the minimum aberration or most preferred designs available for every $k \in [8, 20]$. Figures 12.1 and 12.2 show the aliasing of two-factor interactions for these two sos designs, with generators as specified in Table 12.2. By arranging into columns the interactions in these tables, we conveniently and compactly present the aliasing for each of the embedded designs. These tables enable a practitioner to visualize the additional confusion regarding two-factor interactions that result from adding, e.g., two or three more factors to a 10-factor design.

SOS designs represent a small fraction of all possible resolution IV designs and yet they project to all remaining resolution IV designs. Thus from this subset one can project to all minimum aberration and other good designs. Complete enumeration of

Table 12.2: Generators for SOS Embedded Projection Designs of Size 64

Design	Generators for Factors 7-20 (identified by Yates column number)													
20-14.a	31	39	43	61	49	54	13	21	14	19	25	28	44	58
13-7.b	31	39	43	61	51	62	28							

Design 13-7.b Generators (Yates column number)

31	39	43	61	51	62	28
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Singularity Details (All interactions not listed are clear for designs with $k \leq 13$)

k :	7	8	9	10	11	12	13
		$3*8 =$	$4*9 =$		$5*11$		
	$6*7 =$			$2*10 =$		$1*12$	
		$4*8 =$	$3*9 =$				$11*13$
	$3*4 =$		$8*9 =$				$5*13$
	$2*7 =$			$6*10 =$			$1*13$
	$2*6 =$			$7*10 =$			$12*13$
		$5*8 =$			$3*11 =$		$9*13$
			$5*9 =$		$4*11 =$		$8*13$
	$3*5 =$				$8*11 =$		$4*13$
	$4*5 =$				$9*11 =$		$3*13$
				$1*10 =$		$2*12 =$	$6*13$
	$1*7 =$					$6*12 =$	$2*13$
	$1*6 =$					$7*12 =$	$10*13$
	$1*2 =$					$10*12 =$	$7*13$

Figure 12.1: Design 13-7.b Generators and Aliasing for Embedded Projections

Design 20-14.a Generators (Yates column number)

31	39	43	61	49	54	13	21	14	19	25	28	44	58
Singularity Details						k:	14	15	16	17	18	19	20
1*5 = 6*11 = 8*12 =							3*14 =	7*15 =	2*16 =	4*17 =	13*18 =	10*19 =	9*20
2*7 = 6*10 = 9*12 = 5*13 =							4*14 =		15*16 =	3*17 =	1*18 =	11*19 =	8*20
3*4 = 8*9 = 10*11 = 1*13 =								2*15 =	7*16 =	14*17 =	5*18 =	6*19 =	12*20
6*8 = 11*12 =												9*19 =	10*20
6*9 = 10*12 =												8*19 =	11*20
8*10 = 9*11 =												12*19 =	6*20
1*9 = 8*13 =											12*18 =		5*20
5*8 = 1*12 =											9*18 =		13*20
5*9 = 12*13 =											8*18 =		1*20
3*8 = 4*9 =							12*14 =						17*20
4*12 =							9*14 =			8*17 =			3*20
3*9 = 4*8 =										12*17 =			14*20
7*8 =								12*15 =	9*16 =				2*20
7*9 = 2*12 =									8*16 =				15*20
2*8 =								9*15 =	12*16 =				7*20
2*9 = 7*12 =								8*15 =					16*20
9*10 = 8*11 = 6*12 =													19*20
1*8 = 5*12 = 9*13 =													18*20
3*12 =							8*14 =			9*17 =			4*20
3*6 =							11*14 =			10*17 =		4*19	
3*10 = 4*11 =										6*17 =		14*19	
4*6 =							10*14 =			11*17 =		3*19	
4*10 = 3*11 =							6*14 =					17*19	
7*11 =								6*15 =	10*16 =			2*19	
1*6 = 5*11 =											10*18 =	13*19	
1*10 = 11*13 =											6*18 =	5*19	
5*10 = 6*13 =											11*18 =	1*19	
5*6 = 1*11 = 10*13 =												18*19	
2*6 = 7*10 =									11*16 =			15*19	
6*7 = 2*10 =								11*15 =				16*19	
2*11 =								10*15 =	6*16 =			7*19	
1*2 =								13*15 =	5*16 =		7*18		
1*7 =								5*15 =	13*16 =		2*18		
1*3 = 4*13 =							5*14 =				17*18		
1*4 = 3*13 =										5*17 =	14*18		
3*5 =							1*14 =			13*17 =	4*18		
4*5 =							13*14 =			1*17 =	3*18		
5*7 = 2*13 =								1*15 =			16*18		
2*5 = 7*13 =									1*16 =		15*18		
3*7 =								14*15 =	4*16 =	2*17			
2*3 =								4*15 =	14*16 =	7*17			
2*4 =							7*14 =	3*15 =		16*17			
4*7 =							2*14 =		3*16 =	15*17			

Figure 12.2: Design 20-14.a Generators and Aliasing for Embedded Projections

these projections is prohibitive for large n . However, we have found that naïve projections from sos designs at $n = 64$ and $n = 128$ identify the best resolution IV designs.

It is known from projective geometry that for $n = 16, 32, 64, \dots$, sos designs exist at $k = n/4 + 1$ (Cheng 2002). Furthermore any sos design D with k factors, and n runs

can be doubled by the construction method $\begin{bmatrix} D & D \\ D & -D \end{bmatrix}$ to produce a sos design of size $2k$

factors and $2n$ runs (Cheng 2002). For $k > n/4 + 1$, all sos designs are doubled sos designs. To construct sos designs for $k = n/4 + 1$, see Cheng (2003). Unfortunately, these designs only represent a small fraction of the total sos designs that exist for any given n .

Complementing Cheng's theoretical results, we have determined for $n = 128$ that there exist 88 resolution IV sos designs, 50 with $k \geq n/4 + 1$, and 38 with $k < n/4$. Figure 12.3 summarizes these findings. Naïve projections of these sos designs lead to minimum aberration designs. Table 12.3 lists the length four words resulting from the naïve projections for $k = 24, 22$, and 21 sos designs. Table 12.4 lists the naïve projections for the $k = 25$ sos designs. Table 12.5 lists the naïve projections for $k = 29, 28, 27$, and 26 sos designs. Table 12.6 lists the naïve projections for the top ten sos designs at $k = 33$. Table 12.7 lists the naïve projections for $k = 40, 36, 34$, and 31 sos designs.

We have found 88 sos designs at 14 different values of k at $n = 128$. Four of these sos designs are the minimum aberration design; this occurs at $k = 25, 29, 40$, and 64. It is interesting to note that even some of the less desirable (in terms of wlp) sos designs often project to minimum aberration designs and other good designs. For instance, at $k = 28$, the sos design 28-21.1157 (ranked number 1157 in terms of wlp) naively projects to the

$n = 8$	$n = 16$	$n = 32$	$n = 64$	$n = 128$	k/n
$k = 4$	$k = 8$	$k = 16$	$k = 32$	$k = 64$	$1/2$
<hr/>					
	$k = 5_{(\text{res. } v)}$	$k = 10$	$k = 20$	$k = 40$	$5/16$
		$k = 9$	$k = 18$	$k = 36$	$9/32$
			$k = 17_{(5 \text{ types})}$	$k = 34_{(5 \text{ types})}$	$17/64$
				$k = 33_{(42 \text{ types})}$	$33/128$
					$65/256$
					\vdots
<hr/>					
			$k = 13$	$k = 31$ $k = 29$ $k = 28$ $k = 27$ $k = 26$ $k = 25$ $k = 24$ $k = 22$ $k = 21$	
				} 38 designs	

Note: All sos designs below the dashed line are even/odd designs.

Figure 12.3: Existence of SOS Designs

Table 12.3: $k = 24, 22$, and 21 SOS Designs Naïve Projections Length-4 Words ($w_{4,\dots}$), $n = 128$

k	MA	sos24a	sos24b	sos24c	sos24d	sos24e	sos24f	sos22a	sos22b	sos21a	sos21b	sos21c	sos21d	sos21e
24	102	103	104	109	111	115	115							
23	83	84	85	88	88	92	92							
22	65	68	68	70	68	72	72							
21	51	53	54	53	52	58	56	69	85	52	56	64	80	112
20	36	41	41	41	38	44	42	41	50	40	44	48	60	80
19	27	30	30	31	28	33	30	30	37	30	34	36	44	58
18	20	22	22	23	20**	23	21	23	27	23	25	27	31	41
17	15	15**	15**	16	15**	17	15**	17	18	17	19	19	21	28
16	10	11	11	11	11	12	11	12	12	12	13	12	13	18
15	7	7**	7**	7**	7*	8	7*	8	8	8	8	8	8	12
14	3	4	4	4	3*	5	3*	5	4	5	5	5	4	8
13	2	2*	2*	2*	2**	3	2**	3	2*	3	2*	2*	2*	5
12	1	1**	1**	1**	1**	1**	1**	1**	1**	1**	1**	1**	1**	3
11	0,6	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	1
10	0,3	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*
9	0,0,3	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*

Table 12.5: $k = 29, 28, 27$, and 26 SOS Designs Naïve Projections Length-4 Words (w_4, \dots), $n = 128$

k	MA	sos29a	sos29b	sos29c	sos28	sos27a	sos27b	sos27c	sos27d	sos26a	sos26b
29	266	266*	306	370							
28	210	210*	250	308	290						
27	180	180*	208	254	237		207	210	234		
26	152	152*	173	207	191		163	176	190	181	190
25	124	126	141	167	153		133	145	153	143	146
24	102	102*	114	135	121		105	117	122	113	114
23	83	85	90	107	94		86	92	96	91	95
22	65	69	71	83	71		68	73	76	70	77
21	51	56	56	63	52		53	57	59	54	62
20	36	44	43	48	36*		41	45	44	39	48
19	27	34	32	35	27*		30	34	33	30	37
18	20	25	23	25	20**		22	24	24	22	27
17	15	17	15**	17	15**		16	18	17	16	20
16	10	10*	11	11	11		11	12	12	11	14
15	7	7**	7*	7**	7*		7**	8	8	7**	9
14	3	4	3*	4	3*		4	5	5	4	5
13	2	2*	2**	2*	2**		2*	3	2*	2*	3
12	1	1**	1**	1**	1**		1**	1**	1**	1**	1*
11	0,6	0*	0*	0*	0*		0*	0*	0*	0*	0*
10	0,3	0*	0*	0*	0*		0*	0*	0*	0*	0*
9	0,0,3	0*	0*	0*	0*		0*	0*	0*	0*	0*

Table 12.6: Top Ten $k = 33$ SOS Designs Naïve Projections Length-4 Words (w_4, \dots), $n = 128$

k	MA	sos33a	sos33b	sos33c	sos33d	sos33e	sos33f	sos33g	sos33h	sos33i	sos33j	
33	518	592	592	597	600	600	600	605	605	605	605	
32	452	517	509	517	525	525	521	525	521	525	521	Plus
31	391	447	434	447	453	455	449	457	453	455	445	32
30	335	386	366	386	392	392	386	395	392	392	376	more...
29	266	330	308	330	334	334	330	338	334	334	318	
28	210	280	256	280	285	285	280	289	285	285	266	
27	180	235	210	235	239	240	235	244	239	240	220	
26	152	198	174	198	200	200	198	205	200	200	182	
25	124	165	142	165	165	165	165	169	165	165	149	
24	102	136	113	136	137	137	136	138	137	137	120	
23	83	110	91	112	112	112	110	110	112	112	97	
22	65	90	71	90	90	90	90	90	90	90	76	
21	51	72	53	71	71	72	72	72	72	71	59	
20	36	57	42	56	56	56	57	57	56	56	46	
19	27	45	32	43	43	43	43	43	43	43	34	
18	20	34	24	32	32	32	33	33	32	32	25	
17	15	25	18	23	23	23	24	24	23	23	18	
16	10	17	13	15	15	15	17	17	15	15	12	
15	7	11	8	11	11	11	11	11	11	11	8	
14	3	7	5	7	7	7	7	7	7	7	5	
13	2	3	2*	3	3	3	3	3	3	3	2**	
12	1	2	1**	2	2	2	2	2	2	2	1**	
11	0,6	1	0*	1	1	1	1	1	1	1	0*	
10	0,3	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	
9	0,0,3	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	

Table 12.7: $k = 40, 36, 34$, and 31 SOS Designs Naïve Projections Length-4 Words (w_4, \dots) , $n = 128$

K	MA	sos40	sos36	sos34a	sos34b	sos34c	sos34d	sos34e	sos31a	sos31b
40	1190	1190*								
39	1071	1071*								
38	959	959*								
37	854	854*								
36	756	756*	889							
35	665	665*	776							
34	589	589*	674	616	656	680	720	976		
33	518	518*	582	540	560	588	624	848		
32	452	452*	499	471	480	503	537	733		
31	391	391*	426	408	417	432	458	630	410	439
30	335	335*	360	350	359	366	391	538	345	371
29	266	289	302	300	306	312	330	456	287	310
28	210	248	254	254	261	262	276	384	238	259
27	180	210	213	214	219	222	231	321	195	213
26	152	175	177	177	183	185	190	265	161	176
25	124	145	145	145	150	154	155	217	130	143
24	102	121	117	116	121	126	126	176	105	117
23	83	99	94	95	96	101	100	140	86	94
22	65	79	73	76	78	81	77	109	68	74
21	51	61	59	59	62	63	61	85	55	56
20	36	45	47	44	47	50	48	64	43	44
19	27	36	36	31	36	38	36	46	32	33
18	20	28	27	20*	26	28	26	34	23	24
17	15	21	20	15*	18	20	19	24	16	17
16	10	15	14	11	11	13	13	16	11	11
15	7	10	10	7*	7*	8	9	11	7**	7**
14	3	6	6	3*	3*	5	6	7	4	4
13	2	4	3	2**	2**	2**	3	4	2*	2*
12	1	2	1**	1**	1**	1**	1**	2	1*	1*
11	0,6	1	0*	0*	0*	0*	0*	1	0*	0*
10	0,3	0*	0*	0*	0*	0*	0*	0*	0*	0*
9	0,0,3	0*	0*	0*	0*	0*	0*	0*	0*	0*

minimum aberration design for $k = 20, 19$, and 15 ; and the weak minimum aberration design for $k = 18$ and 17 .

13. Preliminary Results for Resolution IV Designs of Size 256

While identifying almost 300,000 even/odd designs at $n = 128$ was challenging, this pales with the challenge of exhaustively enumerating all designs for $n = 256$ due to the great number of designs. For example, while only 88 sos designs exist at $n = 128$, we have found over 34,000 sos designs in random searches at $n = 256$ (See Table 13.1).

To aid in finding good designs, we implemented a method that combined some of our more successful strategies for finding good designs at $n = 64$ and $n = 128$. Our search at $n = 256$ used two basic approaches. The first approach consists of a random search for sos designs by starting with a design whose columns formed a full factorial and then randomly adding generators to available columns one at a time until an sos design is discovered (stopping if $k > 65$ since all 50 sos designs in this range are already known). Then from these sos designs, we find good designs from the sos designs by naïve projection. The second approach was to find new designs by sequentially building up a factor at a time using t^D as a flawed isomorphic rule to check for isomorphism and retaining the top 2,000 designs from each sequential search and building up from those 2,000 designs.

Table 13.1: Number of Regular Resolution IV⁺ designs

n	# of even/odd sos designs	# of even/odd designs	# of even designs	# of even sos designs
16	1	1	4	1
32	2	5	20	1
64	8	150	349	1
128	87	$\geq 296,960$	$> 10^6$	1
256	$> 34,015$?	?	1

For naïve projection from sos designs approach, there are at least three ways to find sos designs:

- Double the sos designs at $n = 128$
- Random addition of eligible columns until an sos design is found
- Find good designs using software for fixed k and then build up to an sos design

For the sequential buildup technique the issue of which subset of designs to retain at each step is critical. For instance, if only the top 1,000 designs are retained at each buildup step for $n = 256$, then all the designs buildup to sos designs with $k \leq 40$. Future work will explore this issue.

From Franklin (1984) we know the minimum aberration values for designs with up to $k = 17$ factors for $n = 256$. We also know that as early as $k = 11$, we will lose some designs using t^D as a flawed isomorphic rule. However, we still find all the known minimum aberration designs. At $k = 17$, we found 33,142 resolution IV designs with different t^D . Of those, 32,126 are even/odd designs. The 1,016 even designs will continue to grow in number, approximately doubling at each factor until they reach $k = 64$. Based upon our results as $n = 128$, we would expect the number of even/odd designs to increase for each factor until $k = 44$, and then gradually decline at each factor until $k = 80$. (See Table 13.2).

Table 13.2: Existence of Regular Resolution IV⁺ designs

k	# of even/odd designs, $n = 64$	# of even designs, $n = 64$	# of even/odd designs, $n = 128$	# of even designs, $n = 128$	# of e/o designs based on $t^D / G(T2^D)$, $n = 256$	# even designs based on $t^D / G(T2^D)$, $n = 256$
7	2	2	-	-	-	-
8	3	4	2	3	-	-
9	6	6	7	6	3 / 3	3 / 3
10	12	12	19	14	12 / 12	9 / 9
11	20	14	62	30	44 / 50	17 / 24
12	22	21	180	69	153 / 231	44 / 80
13	24	23	487	136	536 / 1,188	89 / 241
14	20	29	1,240	295	1,690 / 6,505	176 / 839
15	15	29	2,926	596	4,668 / 54,269	312 / 3,467
16	11	37	6,208	1,292	12,598 / ?	564 / ?
17	10	30	11,787	2,651	32,126 / ?	1,016 / ?
18	3	30	19,466	5,598	?	?
19	1	24	27,994	11,341	?	?
20	1	23	35,192	22,728	?	?
21	-	16	39,201	43,516	?	?
22	-	15	38,847	79,603	?	?
23	-	9	34,868	?	?	?
24	-	8	28,133	?	?	?
25	-	5	20,569	?	?	?
26	-	4	13,498	?	?	?
27	-	2	8,075	?	?	?
28	-	2	4,284	?	?	?
29	-	1	2,149	?	?	?
30	-	1	976	?	?	?
31	-	1	433	?	?	?
32	-	1	197	?	?	?
33	-	-	101	?	?	?
34	-	-	31	?	?	?
35	-	-	13	?	?	?
36	-	-	8	?	?	?
37	-	-	3	?	?	?
38	-	-	2	?	?	?
39	-	-	1	?	?	?
40	-	-	1	?	?	?

The sheer number of designs that exist at larger n shows the value of the naïve projection method. We are able to rather efficiently evaluate the naïve projections of sos designs at $n = 256$. Table 13.3 below shows the best designs found (based on wlp) for each respective k , and the corresponding alp, number of degrees of freedom used for main effects and two-factor interactions, the number of clear two-factors, and L_{\max} for each design. The Yates ordered columns for those designs are listed in Table 13.4.

We have found over 34,015 sos designs at $n = 256$. The sos designs found occur at $k = 33, \dots, 66, 68, 72, 80$, and 128 at $n = 256$. Future work will involve improving methods of finding good sos designs.

Additional future work will involve looking at ways to refining the naïve projection method to possibly including additional projections. It is no surprise that empirical evidence at $n = 128$ demonstrated at times the second best (or worse) projection for one design, could eventually lead to a better design a few projections later. Consider the even/odd 2_{IV}^{40-33} design. The naïve projections based on minimizing t^D lead to a different design at $k = 16$ than if the criteria looked at only minimizing the length-4 and length-5 words with ties broken arbitrarily. The hope would be to find a method to identify which small set of projections lead to good designs. We would want as small a set of projections as possible that lead to good designs to avoid the combinatorial problem of having to look at all possible combinations of projections.

Table 13.3: Characterization of Good Designs for $n = 256$

k	w_4	w_5	w_6	df	C2FI	L_{\max}	alp
9	0	0	0	45	36	1	36
10	0	0	1	55	45	1	45
11	0	0	6	66	55	1	55
12	0	0	12	78	66	1	66
13	0	3	12	91	78	1	78
14	0	9	18	105	91	1	91
15	0	15	30	120	105	1	105
16	0	24	44	136	120	1	120
17	0	34	68	153	136	1	136
18	3	36	114	162	135	2	135 9
19	4	48	168	178	147	2	147 12
20	5	64	240	195	160	2	160 15
21	9	104	268	206	162	3	162 21 2
22	14	137	346	218	168	3	168 21 7
23	20	172	450	217	136	3	136 57 1
24	27	214	582	221	120	3	120 75 2
25	34	266	752	227	108	3	108 90 4
26	43	325	963	231	94	3	94 102 9
27	53	395	1224	235	80	4	80 114 13 1
28	64	476	1550	239	66	4	66 126 16 3
29	78	579	1908	246	73	3	73 99 45
30	95	686	2340	245	55	4	55 105 50 5
31	113	792	2928	242	21	6	21 140 41 6 1 2
32	133	932	3576	245	19	6	19 124 57 9 2 2
33	153	1095	4360	248	17	6	17 106 75 13 2 2
34	176	1280	5272	252	15	6	15 97 80 21 2 3
35	200	1488	6360	254	9	6	9 88 88 28 2 4

Table 13.3 (Continued)

k	w_4	w_5	w_6	df	C2FI	L_{\max}	alp
36	225	1728	7632	255	0	6	0 81 96 36 0 6
37	264	2004	8928	252	2	8	2 50 102 56 0 2 2 1
38	297	2304	10592	253	1	8	1 33 104 72 0 2 0 3
39	333	2632	12512	254	1	8	1 21 92 96 0 0 2 3
40	370	3008	14720	255	0	8	0 10 80 120 0 0 0 5
41	482	3048	17583	253	10	10	10 25 59 56 39 9 6 5 2 1
42	545	3388	20650	254	10	10	10 24 56 43 45 16 6 9 2 1
43	619	3818	23512	250	0	13	0 22 30 100 27 0 0 23 3 1 0 0 1
44	685	4290	27229	251	0	13	0 17 21 102 39 0 0 12 14 0 1 0 1
45	760	4792	31458	252	0	13	0 16 12 92 59 0 0 6 14 6 0 1 1
46	838	5352	36209	253	0	13	0 16 0 84 79 0 0 2 12 10 2 0 2
47	926	5980	41305	254	0	14	0 16 0 52 110 1 0 2 6 12 5 1 0 2
48	1019	6648	47182	255	0	15	0 16 0 24 132 7 0 2 0 18 0 6 0 0 2
49	1154	7383	52815	253	0	15	0 0 0 36 119 27 0 5 0 0 0 0 6 8 3
50	1257	8200	60044	254	0	15	0 0 0 16 120 46 0 5 0 0 0 0 10 7
51	1365	9100	68068	255	0	15	0 0 0 0 112 70 0 5 0 0 0 0 0 17
52	1500	9264	80976	249	0	24	0 0 0 6 102 32 0 0 0 48 0 8 0 0 0 0 0 0 0 0 0 0 0 1
53	1632	10164	91572	250	0	25	0 0 0 3 81 56 0 0 0 24 24 8 0 0 0 0 0 0 0 0 0 0 0 1
54	1769	11152	103232	251	0	26	0 0 0 1 57 82 0 0 0 8 32 16 0 0 0 0 0 0 0 0 0 0 0 1
55	1911	12240	116000	252	0	27	0 0 0 0 30 110 0 0 0 24 32 0 0 0 0 0 0 0 0 0 0 0 1
56	2058	13440	129920	253	0	28	0 0 0 0 140 0 0 0 0 56 0 0 0 0 0 0 0 0 0 0 0 0 1
57	2282	14280	146272	254	0	28	0 0 0 0 140 0 0 0 0 0 56 0 0 0 0 0 0 0 0 0 0 0 1
58	2534	15120	164304	255	0	29	0 0 0 0 140 0 0 0 0 0 56 0 0 0 0 0 0 0 0 0 0 0 1
59	2870	14256	197856	234	0	22	0 0 0 0 0 48 112 0 0 0 0 0 0 0 0 0 0 0 11 4
60	3075	15552	219840	235	0	22	0 0 0 0 0 0 160 0 0 0 0 0 0 0 0 0 0 0 15
61	3307	16848	244344	236	0	23	0 0 0 0 0 0 0 112 48 0 0 0 0 0 0 0 0 0 0 3 12

Table 13.3 (Continued)

k	w_4	w_5	w_6	df	C2FI	L_{\max}	alp
62	3548	18252	270960	237	0	24	000000007279900000000000069
63	3798	19773	299796	238	0	25	0000000040932700000000000096
64	4057	21420	330960	239	0	26	0000000016905400000000000123
65	4325	23203	364560	240	0	26	000000007090000000000000015
66	4619	24989	401898	241	0	27	0000000042942400000000000213
67	4924	26912	442160	242	0	28	0000000021874840000000000411
68	5240	28982	485484	243	0	29	0000000076972120000000000069
69	5567	31210	532008	244	0	30	00000000409624 $a_{29}=8$ $a_{30}=7$
70	5905	33612	581862	245	0	31	000000000012040 $a_{30}=10$ $a_{31}=5$
71	6273	36014	636851	246	0	32	000000000072808 $a_{31}=11$ $a_{32}=4$
72	6654	38586	695799	247	0	33	00000000003696271 $a_{32}=12$ $a_{33}=3$
73	7048	41343	758875	248	0	34	00000000001288573 $a_{33}=13$ $a_{34}=2$
74	7455	44296	826252	249	0	35	0000000000056986 $a_{34}=14$ $a_{35}=1$
75	7875	47460	898100	250	0	35	00000000000015010 $a_{35}=15$
76	8330	50625	976808	251	0	36	0000000000009070 $a_{36}=15$
77	8800	54000	1060766	252	0	37	000000000000459916 $a_{37}=15$
78	9285	57600	1150184	253	0	38	000000000000159748 $a_{38}=15$
79	9785	61440	1245272	254	0	39	00000000000006496 $a_{39}=15$
80	10300	65536	1346240	255	0	40	0000000000000160 $a_{40}=15$

Table 13.4: Generators for Table 13.3 Designs for $n = 256$

k	Design columns (Yates standard order)
9	1 2 4 8 16 32 64 128 255
10	1 2 4 8 16 32 63 64 128 199
11	1 2 4 8 16 32 64 127 128 143 179
12	1 2 4 8 16 32 64 127 128 143 179 213
13	1 2 4 8 16 32 64 105 127 128 143 179 213
14	1 2 4 8 16 27 32 64 105 127 128 143 179 213
15	1 2 4 8 16 27 32 46 64 105 127 128 143 179 213
16	1 2 4 8 16 32 64 75 85 108 127 128 143 150 179 189
17	1 2 4 8 16 32 64 75 85 108 127 128 143 150 179 189 229
18	1 2 4 8 16 27 32 46 64 92 105 127 128 143 179 182 194 213
19	1 2 4 8 16 27 32 46 64 92 105 127 128 143 179 182 194 213 229
20	1 2 4 8 16 27 32 46 64 92 105 127 128 143 179 182 194 213 229 248
21	1 2 4 8 16 23 27 32 46 64 92 105 127 128 143 173 179 213 217 227 254
22	1 2 4 8 16 27 32 46 64 77 88 105 127 128 143 158 164 179 185 201 213 234
23	1 2 4 7 8 16 27 32 46 64 77 105 127 128 143 158 179 185 201 213 220 228 234
24	1 2 4 7 8 16 27 32 46 64 77 94 105 127 128 143 158 179 185 201 213 220 228 234
25	1 2 4 8 16 27 32 46 64 77 87 105 112 127 128 143 158 166 179 180 185 213 220 232 237
26	1 2 4 8 16 27 32 46 64 77 87 105 112 124 127 128 143 158 166 179 180 185 213 220 232 237
27	1 2 4 8 16 27 32 46 64 77 87 105 112 124 127 128 143 158 166 179 180 185 213 219 220 232 237
28	1 2 4 8 16 27 32 46 64 77 87 105 112 124 127 128 143 158 166 179 180 185 213 219 220 232 237 25
29	1 2 4 8 16 23 27 32 39 46 58 64 77 84 105 124 127 128 143 145 146 179 185 200 213 217 225 228 234
30	1 2 4 8 16 27 32 35 46 64 77 87 88 105 112 127 128 143 158 166 179 180 185 193 210 213 219 220 232 237
31	1 2 4 8 16 32 43 50 62 64 75 78 83 85 88 108 113 118 127 128 138 143 149 150 162 173 179 201 232 239 244
32	1 2 4 8 16 32 43 50 62 64 75 78 83 85 88 108 113 118 127 128 138 140 143 149 150 162 173 179 201 232 239 244
33	1 2 4 8 16 32 43 50 62 64 75 78 83 85 88 108 113 118 127 128 138 140 143 149 150 155 162 173 179 201 232 239 244
34	1 2 4 8 16 23 32 43 50 62 64 75 78 83 85 88 108 113 118 127 128 138 140 143 149 150 162 173 179 185 201 232 239 244
35	1 2 4 8 16 23 32 43 50 62 64 75 78 83 85 88 108 113 118 127 128 138 140 143 149 150 162 173 179 185 201 208 232 239 244
36	1 2 4 8 16 23 32 43 50 62 64 75 78 83 85 88 108 113 118 127 128 138 140 143 149 150 162 173 179 185 201 208 229 232 239 244
37	1 2 4 8 16 27 28 32 35 37 46 55 64 70 77 87 105 106 112 124 127 128 137 143 158 166 179 180 185 202 213 219 220 231 232 237 25
38	1 2 4 8 16 27 28 32 35 37 46 55 64 70 77 87 88 105 106 112 124 127 128 137 143 158 166 179 180 185 202 213 219 220 231 232 237 250
39	1 2 4 8 16 27 28 32 35 37 46 55 64 70 77 87 88 105 106 112 124 127 128 137 143 158 166 179 180 185 193 202 213 219 220 231 232 237 250

Table 13.4 (Continued)

<i>k</i>	Design columns (Yates standard order)																							
40	1 2 4 8 16 27 28 32 35 37 46 55 64 70 77 87 88 105 106 112 124 127 128 137 143 158 166 179 180 185 193 202 210 213 219 220 231 232 237 250																							
41	1 2 4 8 14 16 32 39 42 50 53 57 62 64 67 70 74 76 81 84 87 91 93 128 138 151 157 166 171 177 188 196 199 203 210 216 226 233 239 243 244																							
42	1 2 4 8 14 16 32 39 42 50 53 57 62 64 67 70 74 76 81 84 87 91 93 128 138 151 157 166 171 177 188 196 199 203 210 216 223 226 233 239 243 244																							
43	1 2 4 8 11 16 19 22 32 35 38 49 61 62 64 71 90 93 106 109 114 117 120 127 128 131 141 153 154 159 170 175 181 182 187 193 198 200 207 212 227 250 253																							
44	1 2 4 8 11 16 19 22 32 35 38 49 61 62 64 71 90 93 106 109 114 117 120 127 128 131 141 153 154 159 169 170 175 181 182 187 193 198 200 207 212 227 250 253																							
45	1 2 4 8 11 16 19 22 32 35 38 49 61 62 64 71 90 93 106 109 114 117 120 127 128 131 141 153 154 159 169 170 175 181 182 187 193 198 200 207 211 212 227 250 253																							
46	1 2 4 8 11 16 19 22 32 35 38 49 61 62 64 71 90 93 106 109 114 117 120 127 128 131 141 153 154 159 169 170 175 181 182 187 193 198 200 207 211 212 227 228 250 253																							
47	1 2 4 8 11 16 19 22 32 35 38 49 52 61 62 64 71 90 93 106 109 114 117 120 127 128 131 141 153 154 159 169 170 175 181 182 187 193 198 200 207 211 212 227 228 250 253																							
48	1 2 4 8 11 16 19 22 32 35 38 49 52 61 62 64 71 90 93 106 109 114 117 120 127 128 131 141 153 154 159 169 170 175 181 182 184 187 193 198 200 207 211 212 227 228 250 253																							
49	1 2 4 7 8 13 14 16 21 26 31 32 42 49 50 52 55 56 59 62 64 75 83 88 101 110 118 125 128 133 143 147 150 156 163 166 169 172 176 181 191 193 202 210 217 228 239 247 252																							
50	1 2 4 7 8 13 14 16 21 26 31 32 42 49 50 52 55 56 59 62 64 75 83 88 101 110 118 125 128 133 143 147 150 156 163 166 169 172 176 181 186 191 193 202 210 217 228 239 247 252																							
51	1 2 4 7 8 13 14 16 21 26 31 32 42 47 49 50 52 55 56 59 62 64 75 83 88 101 110 118 125 128 133 143 147 150 156 163 166 169 172 176 181 186 191 193 202 210 217 228 239 247 252																							
52	1 2 4 8 16 21 30 32 37 43 51 54 57 58 60 64 75 78 86 90 92 95 98 103 104 109 117 123 126 128 131 133 137 142 146 159 167 170 173 177 193 198 204 208 215 219 221 222 224 239 243 244																							
53	1 2 4 8 16 21 30 32 37 43 51 54 57 58 60 64 75 78 86 90 92 95 98 100 103 104 109 117 123 126 128 131 133 137 142 146 159 167 170 173 177 193 198 204 208 215 219 221 222 224 239 243 244																							
54	1 2 4 8 16 21 30 32 37 43 51 54 57 58 60 64 75 78 86 90 92 95 98 100 103 104 109 117 123 126 128 131 133 137 142 146 159 167 170 173 177 190 193 198 204 208 215 219 221 222 224 239 243 244																							
55	1 2 4 8 16 21 30 32 37 43 51 54 57 58 60 64 75 78 86 90 92 95 98 100 103 104 109 117 123 126 128 131 133 137 142 146 152 159 167 170 173 177 190 193 198 204 208 215 219 221 222 224 239 243 244																							
56	1 2 4 8 16 21 30 32 37 43 51 54 57 58 60 64 75 78 86 90 92 95 98 100 103 104 109 117 123 126 128 131 133 137 142 146 152 159 167 170 173 177 190 193 198 204 208 215 219 221 222 224 239 243 244 249																							
57	1 2 4 8 16 21 30 32 37 43 51 54 57 58 60 64 75 78 86 90 92 95 98 100 103 104 109 117 123 126 128 131 133 137 142 146 152 159 167 170 173 177 187 190 193 198 204 208 215 219 221 222 224 239 243 244 249																							

Table 13.4 (Continued)

k	Design columns (Yates standard order)
58	1 2 4 8 16 21 30 32 37 43 51 54 57 58 60 64 75 78 86 90 92 95 98 100 103 104 109 117 123 126 128 131 133 137 142 146 152 159 167 170 173 177 187
59	190 193 198 204 208 215 219 221 222 224 229 239 243 244 249
60	1 2 4 8 13 14 16 23 27 32 38 42 44 51 52 56 64 71 81 82 88 93 94 99 100 104 111 112 117 118 121 122 128 131 133 137 140 145 150 154 157 161 164
61	167 168 173 178 185 193 198 205 208 213 220 223 226 233 238 251
62	1 2 4 8 13 14 16 23 27 32 38 42 44 51 52 56 64 71 81 82 88 93 94 99 100 104 111 112 117 118 121 122 128 131 133 137 140 145 150 154 157 161
63	167 168 173 178 185 193 198 205 208 213 220 223 226 233 238 251
64	1 2 4 8 13 14 16 23 27 32 38 42 44 51 52 56 64 71 81 82 88 93 94 99 100 104 111 112 117 118 121 122 128 131 133 137 140 143 145 150 154 157 161
65	167 168 173 178 185 193 198 202 205 208 213 217 220 223 226 233 238 251
66	1 2 4 8 13 14 16 23 27 32 38 42 44 51 52 56 64 71 81 82 88 93 94 99 100 104 111 112 117 118 121 122 128 131 133 137 140 143 145 150 154 157
67	167 168 173 178 181 185 193 198 202 205 208 213 217 220 223 226 233 238 251
68	1 2 4 8 13 14 16 23 27 32 38 41 42 44 51 52 56 64 71 81 82 88 93 94 99 100 104 111 112 117 118 121 122 128 131 133 137 140 143 145 150 154
69	167 168 173 178 181 185 193 198 202 205 208 213 217 220 223 226 233 238 251
70	1 2 4 8 13 14 16 23 27 32 38 41 42 44 51 52 56 63 64 71 81 82 88 93 94 99 100 104 111 112 117 118 121 122 124 128 131 133 137 140 143 145
71	167 168 173 178 181 185 193 198 202 205 208 211 213 217 220 223 226 233 238 251
72	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
73	167 168 173 178 180 183 185 190 193 198 200 203 205 208 211 217 226 228 233 238 241
74	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
75	167 168 173 178 180 183 185 190 193 198 200 203 205 208 211 213 217 226 228 233 238 241
76	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
77	167 168 173 178 180 183 185 190 193 198 200 203 205 208 211 213 217 226 228 233 238 241
78	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
79	167 168 173 178 180 183 185 190 193 198 200 203 205 208 211 213 217 226 228 233 238 241
80	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
81	167 168 173 178 180 183 185 190 193 198 200 203 205 208 211 213 217 226 228 233 238 241
82	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
83	167 168 173 178 180 183 185 190 193 198 200 203 205 208 211 213 217 226 228 233 238 241
84	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
85	167 168 173 178 180 183 185 190 193 198 200 203 205 208 211 213 217 226 228 233 238 241
86	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
87	167 168 173 178 180 183 185 190 193 198 200 203 205 208 211 213 217 226 228 233 238 241
88	1 2 4 8 16 23 25 26 28 32 39 43 45 46 51 53 54 56 64 71 73 74 76 81 82 84 88 95 99 101 102 104 111 112 119 123 125 126 128 131 133 137 142 145 150
89	167 168 173 178 180 183 185 190 193 198 2

Table 13.4 (Continued)

k	Design columns (Yates standard order)																																															
73	1	2	4	8	16	23	25	26	28	32	39	43	45	46	51	53	54	56	63	64	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126	128	131	133	137	142	145			
74	150	152	155	157	161	166	170	172	175	178	180	183	185	190	193	198	200	203	205	208	211	213	217	226	228	233	238	241																				
	1	2	4	8	16	23	25	26	28	32	39	43	45	46	51	53	54	56	63	64	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126	128	131	133	137	142	145			
75	150	152	155	157	161	166	170	172	175	178	180	183	185	190	193	198	200	203	205	208	211	213	217	222	226	228	233	238	241																			
	1	2	4	8	16	23	25	26	28	32	39	43	45	46	51	53	54	56	63	64	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126	128	131	133	137	142	145			
76	150	152	155	157	161	166	170	172	175	178	180	183	185	190	193	198	200	203	205	208	211	213	217	222	226	228	231	233	238	241																		
	1	2	4	8	15	16	23	25	26	28	32	39	43	45	46	51	53	54	56	63	64	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126	128	131	133	137	142	145		
77	145	150	152	155	157	161	166	170	172	175	178	180	183	185	190	193	198	200	203	205	208	211	213	217	222	226	228	231	233	238	241																	
	1	2	4	8	15	16	23	25	26	28	32	39	43	45	46	51	53	54	56	63	64	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126	128	131	133	137	142	145		
78	145	150	152	155	157	161	166	170	172	175	178	180	183	185	190	193	198	200	203	205	208	211	213	217	222	226	228	231	233	238	241	246																
	1	2	4	8	15	16	23	25	26	28	32	39	43	45	46	51	53	54	56	63	64	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126	128	131	133	137	142	145		
79	145	150	152	155	157	161	166	170	172	175	178	180	183	185	190	193	198	200	203	205	208	211	213	217	222	226	228	231	233	238	241	246	250															
	1	2	4	8	15	16	23	25	26	28	32	39	43	45	46	51	53	54	56	63	64	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126	128	131	133	137	142	145		
80	145	150	152	155	157	161	166	170	172	175	178	180	183	185	190	193	198	200	203	205	208	211	213	217	222	226	228	231	233	238	241	246	250	252														
	1	2	4	8	15	16	23	25	26	28	32	39	43	45	46	51	53	54	56	63	64	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126	128	131	133	137	142	145		
	145	150	152	155	157	161	166	170	172	175	178	180	183	185	190	193	198	200	203	205	208	211	213	217	222	226	228	231	233	238	241	246	250	252	255													

14. Conclusions

This dissertation has introduced the alp which provides another useful characterization of designs. The alp of a design contains the number of clear two-factor interactions, the number of degrees of freedom used for main effects and two-factor interactions, and lists the length of the largest set of aliased two-factor interactions. The alp can be used to calculate the number of length-four words, and is helpful in differentiating designs.

We have also studied projections of designs. We now know that all regular resolution IV designs have at least one sos parent. We know an examination of projections from all the sos designs will result in a complete set of regular resolution IV designs. We have introduced a method to find good designs using naïve projections from sos designs instead of an exhaustive search.

We have examined some of the properties of the T matrix and demonstrated its usefulness in searching for good designs. We have found the minimum aberration designs for $n = 128$ based upon our isomorphic conjecture. We list not only these designs and their properties, but provide a catalog of designs with respect to word length pattern, degrees of freedom used, clear two-factor interactions, and minimizing the length of the longest set of aliased two-factor interactions.

We know that the naïve projections from sos designs leads to all the minimum aberration values for $n = 32, 64$, and 128 . We know that the number of regular resolution IV designs increases at a rate that makes exhaustive searches infeasible beyond $n = 128$ using current technology. We know that projections from the doubled sos design at $k = (5/16)n$ results in excellent (and very often minimum aberration) designs. We

provide a number of interesting designs at $n = 128$ that are alike in several (sometimes all) characterization criteria, yet non-isomorphic.

Finally, we have found over 34,015 sos designs for $n = 256$. We show how the magnitude of the number of designs increases with larger n . We use naïve projections and build up using the best 2,000 designs to provide a preliminary table of the best designs at $n = 256$.

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Appendices

Appendix A: Yates Column Order Design Matrix

Yates Column Order Generator Matrix, For $r > 129, \dots, 255$ $i_r = i_{128} + i_{r-128}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0
0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1
1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0
1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Appendix B: Catalog of Even/Odd Resolution IV Design for $n = 64$

Even-Odd Resolution IV Designs of Size 64

Design	csw #	Generators	d.f.	w ₄ , w ₅ , w ₆ , ...	Alias Length Pattern	E/O Proj.
20-14.a	1	7, 11, 13, 14, 19, 21, 22, 35, 37, 38, 57, 58, 60, 63	63	125, 256, 480, ...	0, 0, 0, 40, 0, 0, 0, 0, 3	a
19-13.a	1	7, 11, 13, 14, 19, 21, 22, 35, 37, 38, 57, 58, 60	62	100, 192, ...	0, 0, 16, 24, 0, 0, 0, 0, 3	a, b
18-12.a	1	7, 11, 13, 14, 19, 21, 22, 35, 37, 57, 58, 60	61	78, 144, ...	0, 3, 25, 12, 0, 0, 0, 3	a, c
18-12.b	2	7, 11, 13, 14, 19, 21, 22, 35, 37, 38, 57, 58	61	84, 128, ...	0, 16, 0, 24, 0, 0, 0, 2, 1	c, h
18-12.c	3	7, 11, 13, 14, 19, 21, 22, 25, 26, 35, 60, 63	63	92, 112, ...	0, 30, 0, 0, 14, 0, 0, 1	f, i
17-11.a	1	7, 11, 13, 14, 19, 21, 35, 37, 57, 58, 60	60	59, 108, ...	0, 9, 27, 4, 0, 0, 3	a, c
17-11.b	2	7, 11, 19, 29, 37, 41, 47, 49, 55, 59, 62	63	60, 80, ...	16, 0, 0, 30	b
17-11.c	*	7, 11, 13, 14, 19, 21, 22, 35, 37, 57, 58	60	64, 96, ...	2, 14, 12, 12, 0, 0, 2, 1	c, f, g, i
17-11.d	3	7, 11, 13, 19, 21, 25, 35, 37, 41, 49, 63	63	65, 75, ...	16, 0, 15, 0, 15	b, d
17-11.e	4	7, 11, 13, 14, 19, 21, 25, 35, 37, 41, 63	63	68, 72, ...	16, 6, 0, 18, 0, 6	d, h
17-11.f	*	7, 11, 13, 14, 19, 21, 22, 25, 35, 60, 63	62	68, 88, ...	4, 26, 0, 0, 12, 2, 0, 1	e, g, j
17-11.g	5	7, 11, 13, 14, 19, 21, 22, 25, 35, 37, 63	63	73, 67, ...	19, 0, 12, 0, 12, 0, 3	h, i, j
17-11.h	7	7, 11, 13, 14, 19, 21, 22, 35, 37, 38, 57	60	76, 64, ...	16, 0, 0, 24, 0, 0, 0, 3	i,
17-11.i	10	7, 11, 13, 14, 19, 21, 22, 25, 26, 35, 60	62	84, 56, ...	16, 14, 0, 0, 0, 14, 0, 1	j, k
17-11.j	6	7, 11, 13, 14, 19, 21, 22, 25, 26, 28, 63	63	105, 35, ...	31, 0, 0, 0, 0, 0, 15	k
16-10.a	1	7, 11, 13, 19, 21, 35, 37, 57, 58, 60	59	43, 81, ...	0, 18, 22, 0, 0, 3	a, b, d
16-10.b	2	7, 11, 19, 29, 37, 41, 47, 49, 55, 59	61	45, 60, ...	15, 0, 15, 15	c

Design	csw #	Generators	d.f.	w ₄ , w ₅ , w ₆ , ...	Alias Length Pattern	E/O Proj.
16-10.c	7	7 11 13 14 19 21 35 37 57 58	59	47, 72, ...	4, 15, 17, 4, 0, 2, 1	d, h, j, l
16-10.d	3	7 11 13 19 21 25 35 37 41 63	61	49, 56, ...	15, 6, 9, 9, 6	c, f, i
16-10.e	8	7 11 13 14 19 21 25 35 60 63	61	49, 68, ...	8, 22, 0, 9, 5, 0, 1	e, g, j, m
16-10.f	9	7 11 13 14 19 21 22 35 57 60	59	51, 64, ...	4, 24, 0, 12, 0, 1, 2	h, n
16-10.g	*	7 11 13 14 19 21 22 35 57 58	57	52, 64, ...	0, 26, 0, 12, 0, 2, 0, 1	j, n
16-10.h	4	7 11 13 14 19 21 25 35 37 63	61	53, 52, ...	18, 3, 9, 9, 3, 3	i, k, l, m
16-10.i	10	7 11 13 14 19 21 22 35 37 57	58	57, 48, ...	15, 0, 12, 12, 0, 0, 3	l, n
16-10.j	5	7 11 13 14 19 21 22 25 35 60	60	61, 44, ...	17, 12, 0, 0, 12, 2, 1	m, n, o
16-10.k	6	7 11 13 14 19 21 22 25 26 60	60	77, 28, ...	29, 0, 0, 0, 0, 14, 1	o
15-9.a	1	7 11 19 30 37 41 49 60 63	58	30, 60, ...	0, 30, 10, 0, 3	a
15-9.b	2	7 11 19 29 30 37 41 49 60	58	30, 61, ...	0, 30, 10, 0, 3	a, c
15-9.c	3	7 11 19 29 37 41 47 49 55	59	33, 44, ...	14, 6, 17, 7	d, g
15-9.d	6	7 11 13 19 21 35 37 57 58	58	33, 54, ...	6, 19, 15, 0, 2, 1	a, c, h, j, n
15-9.e	7	7 11 13 19 21 25 35 60 63	60	34, 52, ...	12, 18, 5, 9, 0, 1	b, e, f, j, o
15-9.f	8	7 11 13 19 21 35 41 49 63	59	35, 42, ...	14, 11, 8, 10, 1	d, i, k
15-9.g	*	7 11 13 14 19 21 41 54 63	60	35, 50, ...	12, 18, 8, 3, 3, 1	e, m, q
15-9.h	*	7 11 13 14 19 21 35 57 60	58	36, 48, ...	8, 20, 8, 4, 1, 2	h, l, m, r
15-9.i	9	7 11 13 19 21 25 35 37 63	59	37, 40, ...	17, 6, 11, 7, 3	g, i, k, n, o

Design	csw #	Generators	d.f.	w ₄ , w ₅ , w ₆ , ...	Alias Length Pattern	E/O Proj.
15-9.j	*	7 11 13 14 19 21 35 57 58	56	37, 48, ...	4, 22, 8, 4, 2, 0, 1	j, m, p, r
15-9.k	4	7 11 13 14 19 21 35 41 63	59	39, 38, ...	19, 2, 16, 2, 4, 1	k, q, r
15-9.l	*	7 11 13 14 19 21 35 37 57	56	41, 36, ...	14, 3, 17, 4, 0, 3	n, r
15-9.m	10	7 11 13 14 19 21 25 35 60	58	43, 34, ...	18, 10, 0, 9, 5, 1	o, q, r, s
15-9.n	*	7 11 13 14 19 21 22 35 57	56	45, 32, ...	14, 12, 0, 12, 0, 2, 1	r, t
15-9.o	5	7 11 13 14 19 21 22 25 58	57	55, 22, ...	27, 0, 0, 0, 12, 3	s, t
14-8.a	1	7 11 19 30 37 41 49 60	57	22, 40, 36, ...	8, 26, 6, 2, 1	d, h, o
14-8.b	2	7 11 19 29 30 37 41 47	59	22, 40, 41, ...	16, 14, 14, 0, 1	a, i, m
14-8.c	6	7 11 19 29 30 37 41 49	57	22, 41, ...	8, 26, 6, 2, 1	d, i, l, o
14-8.d	7	7 11 19 30 37 41 52 56	57	23, 32, ...	13, 15, 12, 3	c, f
14-8.e	8	7 11 13 19 21 41 54 63	59	23, 38, ...	16, 17, 8, 3, 1	a, e, h, k, p
14-8.f	9	7 11 13 19 21 46 54 56	59	23, 40, ...	16, 17, 8, 3, 1	e, i, q
14-8.g	10	7 11 19 29 37 41 47 49	57	24, 31, ...	16, 9, 15, 3	f, m, o
14-8.h	*	7 11 13 19 21 35 57 60	57	24, 36, ...	12, 19, 9, 1, 2	d, g, k, s
14-8.i	*	7 11 13 19 21 41 49 63	57	25, 30, ...	16, 12, 9, 6	f, j, l, p, q
14-8.j	*	7 11 13 19 21 35 57 58	55	25, 36, ...	8, 21, 9, 2, 0, 1	h, i, k, r, s
14-8.k	*	7 11 13 19 21 35 41 63	57	26, 29, ...	18, 8, 12, 4, 1	f, j, p, s
14-8.l	*	7 11 13 14 19 37 57 63	57	26, 32, ...	12, 24, 0, 4, 3	g, t

Design	CSW #	Generators	d.f.	w ₄ , w ₅ , w ₆ , ...	Alias Length Pattern	E/O Proj.
14-8.m	*	7 11 13 14 19 35 53 57	55	27, 32, ...	8, 26, 0, 5, 1, 1	k, t, u
14-8.n	*	7 11 13 19 21 35 37 57	54	28, 27, ...	13, 9, 15, 0, 3	l, s
14-8.o	3	7 11 13 19 21 25 35 60	56	29, 26, ...	19, 8, 5, 9, 1	m, p, q, s, v
14-8.p	*	7 11 13 14 19 35 53 54	51	29, 32, ...	0, 30, 0, 6, 0, 0, 1	r, u
14-8.q	*	7 11 13 14 19 21 41 54	56	30, 25, ...	19, 8, 8, 3, 4	p, t, w, x
14-8.r	*	7 11 13 14 19 21 35 57	54	31, 24, ...	15, 10, 8, 4, 2, 1	s, t, u, w
14-8.s	4	7 11 13 14 19 21 25 54	54	38, 17, ...	25, 0, 0, 9, 6	v, w, x
14-8.t	5	7 11 13 14 19 21 22 57	54	39, 16, ...	25, 0, 0, 12, 0, 3	w
13-7.a	1	7 11 21 25 38 58 60	58	14, 28, ...	20, 18, 6, 1	b, e, g, i
13-7.b	2	7 11 13 30 46 49 63	63	14, 33, ...	36, 0, 14	a, h
13-7.c	3	7 11 19 29 37 59 62	55	15, 24, ...	12, 27, 0, 3	f
13-7.d	4	7 11 19 29 37 41 60	56	15, 27, ...	16, 21, 4, 2	c, g, k, m
13-7.e	5	7 11 13 19 46 49 63	58	15, 28, ...	22, 15, 6, 2	b, g, h, j, l
13-7.f	6	7 11 19 30 37 41 52	55	16, 22, ...	17, 15, 9, 1	d, f, i, m
13-7.g	7	7 11 13 19 37 57 63	56	16, 24, ...	18, 18, 4, 3	c, e, p
13-7.h	8	7 11 19 37 41 60 63	54	16, 26, ...	12, 23, 5, 0, 1	g, k, n
13-7.i	*	7 11 19 29 30 37 41	54	16, 28, ...	12, 23, 5, 0, 1	g, m, o
13-7.j	*	7 11 13 19 37 49 63	55	17, 21, ...	19, 12, 9, 2	d, i, k, l, p

Design	csw #	Generators	d.f.	w ₄ , w ₅ , w ₆ , ...	Alias Length Pattern	E/O Proj.
13-7.k	*	7 11 13 19 35 53 57	54	17, 24, ...	19, 12, 9, 2	e, g, j, n, p, q
13-7.l	*	7 11 19 30 37 41 49	52	18, 20, 24, ...	12, 18, 6, 3	k, m
13-7.m	9	7 11 19 29 37 41 47	54	18, 20, 28, ...	20, 6, 14, 1	i, m, r
13-7.n	10	7 11 13 19 35 49 63	55	18, 21, 24, ...	21, 8, 12, 0, 1	f, l, q
13-7.o	*	7 11 19 29 37 41 49	52	18, 21, 24, ...	12, 18, 6, 3	m
13-7.p	*	7 11 13 19 21 41 54	54	19, 19, ...	20, 9, 8, 4	i, k, l, p, t, n
13-7.q	*	7 11 13 19 21 46 54	54	19, 20, ...	20, 9, 8, 4	l, u
13-7.r	*	7 11 13 19 35 53 54	50	19, 24, ...	6, 24, 6, 0, 0, 1	s, q, o, n
13-7.s	*	7 11 13 19 21 35 57	52	20, 18, ...	16, 11, 9, 2, 1	k, m, p, q, t
13-7.t	*	7 11 13 14 19 37 57	52	22, 16, ...	16, 16, 0, 5, 2	p, v
13-7.u	*	7 11 13 14 19 35 53	50	23, 16, ...	12, 18, 0, 6, 0, 1	q, v
13-7.v	*	7 11 13 19 21 25 46	51	25, 13, ...	23, 0, 5, 10	r, t, u
13-7.w	*	7 11 13 14 19 21 57	51	26, 12, ...	23, 0, 8, 4, 3	t, v
13-7.x	*	7 11 13 14 19 21 54	51	26, 13, ...	23, 0, 8, 4, 3	u, v
12-6.a	1	7 11 29 45 51 62	62	6, 24, ...	36, 12, 2	a
12-6.b	2	7 11 21 46 54 56	57	8, 20, ...	27, 15, 3	a, c, d, e
12-6.c	3	7 11 21 41 51 63	55	9, 18, ...	24, 15, 4	c, h
12-6.d	4	7 11 21 41 54 56	53	10, 15, ...	21, 15, 5	b, d, h

Design	CSW #	Generators	d.f.	w ₄ , w ₅ , w ₆ , ...	Alias Length Pattern	E/O Proj.
12-6.e	6	7 11 19 37 57 63	53	10, 16, 12, ...	20, 18, 2, 1	c, g, j
12-6.f	7	7 11 19 29 37 59	53	10, 16, 16, ...	20, 18, 2, 1	d, f, l
12-6.g	8	7 11 19 29 37 57	53	10, 18, ...	20, 18, 2, 1	c, e, h, i, j, l
12-6.h	5	7 11 13 30 46 49	56	10, 20, ...	30, 6, 8	a, e, k, m
12-6.i	9	7 11 21 25 38 58	52	11, 14, ...	21, 12, 7	d, g, h, o
12-6.j	*	7 11 19 37 57 60	51	11, 16, ...	16, 21, 0, 2	e, j
12-6.k	*	7 11 19 37 41 60	50	12, 13, ...	17, 15, 5, 1	h, j, n, o
12-6.l	10	7 11 13 19 46 49	52	12, 14, 12, ...	23, 9, 7, 1	d, h, j, k, p, r
12-6.m	*	7 11 19 29 37 41	50	12, 14, 12, ...	17, 15, 5, 1	h, l, o
12-6.n	*	7 11 19 37 57 58	49	12, 16, ...	12, 23, 1, 0, 1	i, j, r
12-6.o	*	7 11 19 29 30 37	49	12, 20, ...	12, 23, 1, 0, 1	i, l, s
12-6.p	*	7 11 13 19 37 57	50	13, 12, ...	19, 12, 5, 2	g, h, j, p
12-6.q	*	7 11 13 19 35 53	48	14, 12, ...	15, 14, 6, 0, 1	j, l, p, q
12-6.r	*	7 11 21 25 31 45	48	15, 10, ...	21, 0, 15	o
12-6.s	*	7 11 19 29 30 35	43	15, 16, ...	0, 30, 0, 0, 0, 1	q, s
12-6.t	*	7 11 13 19 21 57	48	16, 9, ...	21, 3, 9, 3	n, o, p
12-6.u	*	7 11 13 19 21 46	48	16, 10, ...	21, 3, 9, 3	r, p, o
12-6.v	*	7 11 13 14 19 53	48	18, 8, ...	21, 8, 0, 6, 1	p, t

Design	csw #	Generators	d.f.	w_4, w_5, w_6, \dots	Alias Length Pattern	E/O Proj.
11-5.a	1	7 11 29 45 51	55	4, 14, ...	34, 9, 1	a, c, f
11-5.b	2	7 25 42 52 63	51	5, 10, ...	25, 15	b
11-5.c	3	7 11 29 46 49	52	5, 12, ...	28, 12, 1	a, b, d, e
11-5.d	4	7 11 21 46 56	50	6, 10, ...	25, 12, 2	b, c, e, h
11-5.e	5	7 11 29 45 49	50	6, 12, 4, ...	25, 12, 2	a, e, f
11-5.f	6	7 11 19 29 62	51	6, 12, 8, ...	27, 12, 0, 1	c, j
11-5.g	7	7 11 21 38 57	48	7, 8, ...	22, 12, 3	b, g
11-5.h	8	7 11 21 41 51	48	7, 9, ...	22, 12, 3	b, e, g, h
11-5.i	*	7 11 19 29 45	48	7, 12, ...	21, 15, 0, 1	d, e, f, i, j
11-5.j	*	7 11 19 37 57	46	8, 8, ...	18, 15, 1, 1	e, g, i
11-5.k	9	7 11 13 30 49	49	8, 10, 4, ...	28, 3, 7	c, e, k, l
11-5.l	*	7 11 19 29 37	46	8, 10, 4, ...	18, 15, 1, 1	e, h, j
11-5.m	10	7 11 13 30 46	49	8, 14, ...	28, 3, 7	f, l
11-5.n	*	7 11 21 25 63	45	9, 6, ...	19, 9, 6	g
11-5.o	*	7 11 21 25 45	45	9, 7, ...	19, 9, 6	g, h
11-5.p	*	7 11 13 19 53	45	10, 6, ...	21, 6, 6, 1	g, h, i, k
11-5.q	*	7 11 19 29 35	42	10, 8, 0, ...	10, 20, 0, 0, 1	i, j
11-5.r	*	7 11 13 19 46	45	10, 8, 4, ...	21, 6, 6, 1	j

Design	CSW#	Generators	d.f.	w ₄ , w ₅ , w ₆ , ...	Alias Length Pattern	E/O Proj.
11-5.s	*	7 11 19 29 30	42	10, 16, ...	10, 20, 0, 0, 1	k
11-5.t	*	7 11 13 14 51	45	14, 4, ...	27, 0, 0, 7	h, i, l
10-4.a	1	7 27 43 53	49	2, 8, ...	33, 6	a, c
10-4.b	2	7 25 42 53	46	3, 6, ...	27, 9	a, b
10-4.c	3	7 11 29 51	47	3, 7, ...	30, 6, 1	a, e, f
10-4.d	4	7 11 29 46	47	3, 8, ...	30, 6, 1	a, f
10-4.e	5	7 11 29 49	44	4, 6, ...	24, 9, 1	a, b, c, d, f
10-4.f	6	7 11 29 45	44	4, 8, ...	24, 9, 1	c, f
10-4.g	8	7 11 21 57	42	5, 4, ...	21, 9, 2	b, d
10-4.h	9	7 11 21 45	42	5, 5, ...	21, 9, 2	b, d, e, f
10-4.i	*	7 11 19 45	40	6, 4, ...	17, 12, 0, 1	d, f
10-4.j	*	7 11 19 29	40	6, 8, ...	17, 12, 0, 1	f
10-4.k	*	7 11 13 51	41	7, 3, ...	24, 0, 7	d, e
10-4.l	*	7 11 13 30	41	7, 7, ...	24, 0, 7	f
9-3.a	1	7 27 45	42	1, 4, ...	30, 3	a, c
9-3.b	2	7 25 43	39	2, 3, ...	24, 6	a, b, c
9-3.c	3	7 27 43	39	2, 4, ...	24, 6	c
9-3.d	6	7 11 53	37	3, 2, ...	21, 6, 1	b, c

Design	csw #	Generators	d.f.	w_4, w_5, w_6, \dots	Alias Length Pattern	E/O Proj.
9-3.e	7	7 11 51	37	3, 3, ...	21, 6, 1	c
9-3.f	8	7 11 29	37	3, 4, ...	21, 6, 1	c
8-2.a	1	15 51	36	0, 2, 1, ...	28	-
8-2.b	*	7 57	33	1, 1, ...	22, 3	-
8-2.c	*	7 27	33	1, 2, ...	22, 3	-

Appendix C: Catalog of Designs, $n = 128$

k = 8, Designs sorted based on word length pattern

Design	wlp	(w ₄ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank			
8-1.1	0	0	0	1	28	0	0	36	28	1	1	55.0998	1		
8-1.2	0	0	0	1	0	2	28	0	0	36	28	1	2	55.0998	2
8-1.3	0	0	1	0	0	3	28	0	0	36	28	1	3	55.0999	3
8-1.4	0	1	0	0	0	4	28	0	0	36	28	1	4	55.1007	4
8-1.5	1	0	0	0	0	5	22	3	0	33	22	2	5	55.1082	5

k = 8, Design generators

Design	Design Generators
8-1.1	127
8-1.2	63
8-1.3	121
8-1.4	15
8-1.5	7

k = 9, Designs sorted based on word length pattern

Design	wlp (w ₄ ,...)			wlp rank			alp	df	C2FI	Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank
9-2.1	0	0	3	0	0	0	0	0	0	36	1	1	1	49.5901	1
9-2.2	0	1	1	1	0	0	0	0	45	36	1	2	2	49.5908	2
9-2.3	0	2	0	0	1	0	0	0	45	36	1	3	3	49.5915	3
9-2.4	0	2	1	0	0	0	0	0	45	36	1	4	4	49.5916	4
9-2.5	1	0	0	2	0	0	0	0	42	30	2	5	5	49.5974	5
9-2.6	1	0	1	0	1	0	0	0	42	30	2	6	6	49.5975	6
9-2.7	1	0	2	0	0	0	0	0	42	30	2	7	7	49.5976	7
9-2.8	1	1	0	0	0	1	0	0	42	30	2	8	8	49.5982	8
9-2.9	1	1	0	1	0	0	0	0	42	30	2	9	9	49.5982	9
9-2.10	1	2	0	0	0	0	0	0	42	30	2	10	10	49.5991	10
9-2.11	2	0	0	0	1	0	0	0	39	24	2	11	11	49.6049	11
9-2.12	2	0	1	0	0	0	0	0	39	24	2	12	12	49.6050	12
9-2.13	3	0	0	0	0	0	0	0	37	21	3	13	13	49.6125	13

k = 9, Design generators

Design	Design Generators
9-2.1	31 121
9-2.2	15 121
9-2.3	15 120
9-2.4	15 51
9-2.5	7 123
9-2.6	7 121
9-2.7	7 59
9-2.8	7 120
9-2.9	7 57
9-2.10	7 27
9-2.11	7 112
9-2.12	7 25
9-2.13	7 11

k = 10, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	rank	CD2*	CD2 rank
10-3.1	0 3 3	1	45	0	0	55	45	1	1	1	44.6334	1
10-3.2	0 4 2	2	45	0	0	55	45	1	2	2	44.6340	2
10-3.3	1 0 6	3	39	3	0	52	39	2	3	3	44.6381	3
10-3.4a	1 2 2	4	39	3	0	52	39	2	4	4	44.6393	4
10-3.4b	1 2 2	4	39	3	0	52	39	2	4	4	44.6393	4
10-3.6	1 3 1	6	39	3	0	52	39	2	6	6	44.6400	6
10-3.7	1 3 2	7	39	3	0	52	39	2	7	7	44.6401	7
10-3.8	1 4 0	8	39	3	0	52	39	2	8	8	44.6407	8
10-3.9	1 4 1	9	39	3	0	52	39	2	9	9	44.6408	9
10-3.10	1 4 2	10	39	3	0	52	39	2	10	10	44.6408	10
10-3.11a	2 0 4	11	33	6	0	49	33	2	11	11	44.6448	11
10-3.11b	2 0 4	11	33	6	0	49	33	2	11	11	44.6448	11
10-3.13	2 1 1	13	33	6	0	49	33	2	13	13	44.6453	13
10-3.14	2 2 0	14	33	6	0	49	33	2	14	14	44.6460	14
10-3.15	2 2 1	15	33	6	0	49	33	2	15	15	44.6461	15
10-3.16	2 3 1	16	33	6	0	49	33	2	16	16	44.6468	16
10-3.17	2 4 0	17	33	6	0	49	33	2	17	17	44.6475	17
10-3.18	3 0 0	18	30	6	1	47	30	3	18	18	44.6513	18
10-3.19	3 0 2	19	30	6	1	47	30	3	19	19	44.6514	19
10-3.20	3 0 2	19	27	9	0	46	27	2	26	26	44.6514	19

k = 10, Designs sorted based on degrees of freedom used

Design	wlp(w_1, \dots)			wlp rank	alp	df	C2FI	Lmax	df rank	C2FI	Lmax	CD2*	CD2 rank		
10-3.1	0	3	3	1	45	0	0	0	0	55	45	1	1	44.6334	1
10-3.2	0	4	2	2	45	0	0	0	0	55	45	1	2	44.6340	2
10-3.3	1	0	6	3	39	3	0	0	0	52	39	2	3	44.6381	3
10-3.4b	1	2	2	4	39	3	0	0	0	52	39	2	4	44.6393	4
10-3.4a	1	2	2	4	39	3	0	0	0	52	39	2	4	44.6393	4
10-3.6	1	3	1	6	39	3	0	0	0	52	39	2	6	44.6400	6
10-3.7	1	3	2	7	39	3	0	0	0	52	39	2	7	44.6401	7
10-3.8	1	4	0	8	39	3	0	0	0	52	39	2	8	44.6407	8
10-3.9	1	4	1	9	39	3	0	0	0	52	39	2	9	44.6408	9
10-3.10	1	4	2	10	39	3	0	0	0	52	39	2	10	44.6408	10
10-3.11b	2	0	4	11	33	6	0	0	0	49	33	2	11	44.6448	11
10-3.11a	2	0	4	11	33	6	0	0	0	49	33	2	11	44.6448	11
10-3.13	2	1	1	13	33	6	0	0	0	49	33	2	13	44.6453	13
10-3.14	2	2	0	14	33	6	0	0	0	49	33	2	14	44.6460	14
10-3.15	2	2	1	15	33	6	0	0	0	49	33	2	15	44.6461	15
10-3.16	2	3	1	16	33	6	0	0	0	49	33	2	16	44.6468	16
10-3.17	2	4	0	17	33	6	0	0	0	49	33	2	17	44.6475	17
10-3.18	3	0	0	18	30	6	1	0	0	47	30	3	18	44.6513	18
10-3.19	3	0	2	19	30	6	1	0	0	47	30	3	19	44.6514	19
10-3.21	3	0	3	21	30	6	1	0	0	47	30	3	20	44.6515	21

k = 10, Designs sorted based on minimizing Lmax

Design	wlp(w ₁ ,...) wlp			alp	df	C2FI	Lmax	df	C2FI	Lmax	rank	CD2*	CD2
													rank
10-3.1	0	3	3	1	45	0	0	0	55	45	1	44.6334	1
10-3.2	0	4	2	2	45	0	0	0	55	45	2	44.6340	4
10-3.3	1	0	6	3	39	3	0	0	52	39	3	44.6381	3
10-3.4b	1	2	2	4	39	3	0	0	52	39	4	44.6393	5
10-3.4a	1	2	2	4	39	3	0	0	52	39	4	44.6393	4
10-3.6	1	3	1	6	39	3	0	0	52	39	6	44.6400	6
10-3.7	1	3	2	7	39	3	0	0	52	39	7	44.6401	7
10-3.8	1	4	0	8	39	3	0	0	52	39	8	44.6407	8
10-3.9	1	4	1	9	39	3	0	0	52	39	9	44.6408	9
10-3.10	1	4	2	10	39	3	0	0	52	39	10	44.6408	10
10-3.11b	2	0	4	11	33	6	0	0	49	33	11	44.6448	11
10-3.11a	2	0	4	11	33	6	0	0	49	33	11	44.6448	11
10-3.13	2	1	1	13	33	6	0	0	49	33	13	44.6453	13
10-3.14	2	2	0	14	33	6	0	0	49	33	14	44.6460	14
10-3.15	2	2	1	15	33	6	0	0	49	33	15	44.6461	15
10-3.16	2	3	1	16	33	6	0	0	49	33	16	44.6468	16
10-3.17	2	4	0	17	33	6	0	0	49	33	17	44.6475	17
10-3.20	3	0	2	19	27	9	0	0	46	27	18	44.6514	19
10-3.22	3	0	3	21	27	9	0	0	46	27	19	44.6515	21
10-3.24	3	0	4	23	27	9	0	0	46	27	20	44.6516	23

k = 10, Design generators

Design	Design Generators		
10-3.1	15	51	121
10-3.2	15	51	120
10-3.3	7	59	93
10-3.4a	7	27	109
10-3.4b	7	57	90
10-3.6	7	27	121
10-3.7	7	27	120
10-3.8	7	27	101
10-3.9	7	27	99
10-3.10	7	27	45
10-3.11a	7	26	121
10-3.11b	7	59	112
10-3.13	7	25	106
10-3.14	7	27	112
10-3.15	7	25	120
10-3.16	7	25	43
10-3.17	7	51	112
10-3.18	7	11	125
10-3.19	7	121	122
10-3.20	7	112	121
10-3.21	7	11	115
10-3.22	7	25	97
10-3.24	7	25	42

k = 11, Designs sorted based on word length pattern

Design	wlp(w _i ,...)		wlp rank	alp		df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
						rank	rank	rank	rank	rank	rank	rank	rank
11-4.1	0	6	6	1	55	0	0	0	0	66	55	1	1
11-4.2	1	4	6	2	49	3	0	0	0	63	49	2	2
11-4.3	1	5	6	3	49	3	0	0	0	63	49	2	3
11-4.4	1	6	4	4	49	3	0	0	0	63	49	2	4
11-4.5	1	6	5	5	49	3	0	0	0	63	49	2	5
11-4.6	1	6	6	6	49	3	0	0	0	63	49	2	6
11-4.7	1	7	4	7	49	3	0	0	0	63	49	2	7
11-4.8	2	0	12	8	43	6	0	0	0	60	43	2	8
11-4.9a	2	4	4	9	43	6	0	0	0	60	43	2	9
11-4.9b	2	4	4	9	43	6	0	0	0	60	43	2	9
11-4.9c	2	4	4	9	43	6	0	0	0	60	43	2	9
11-4.12	2	5	4	12	43	6	0	0	0	60	43	2	12
11-4.13a	2	6	2	13	43	6	0	0	0	60	43	2	13
11-4.13b	2	6	2	13	43	6	0	0	0	60	43	2	13
11-4.15	2	6	3	15	43	6	0	0	0	60	43	2	15
11-4.16	2	6	4	16	43	6	0	0	0	60	43	2	16
11-4.17	2	8	4	17	43	6	0	0	0	60	43	2	17
11-4.18	3	0	10	18	37	9	0	0	0	57	37	2	18
11-4.19	3	0	11	19	40	6	1	0	0	58	40	3	19
11-4.20	3	2	4	20	37	9	0	0	0	57	37	2	20

k = 11, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
11-4.1	0 6 6	1	55 0 0 0 0	66	55	1	1	1	1	40.1723	1
11-4.2	1 4 6	2	49 3 0 0 0	63	49	2	2	2	2	40.1771	2
11-4.3	1 5 6	3	49 3 0 0 0	63	49	2	3	3	3	40.1778	3
11-4.4	1 6 4	4	49 3 0 0 0	63	49	2	4	4	4	40.1783	4
11-4.5	1 6 5	5	49 3 0 0 0	63	49	2	5	5	5	40.1784	5
11-4.6	1 6 6	6	49 3 0 0 0	63	49	2	6	6	6	40.1784	6
11-4.7	1 7 4	7	49 3 0 0 0	63	49	2	7	7	7	40.1789	7
11-4.8	2 0 12	8	43 6 0 0 0	60	43	2	8	8	8	40.1809	8

k = 11, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
11-4.1	0 6 6	1	55 0 0 0 0	66	55	1	1	1	1	40.1723	1
11-4.2	1 4 6	2	49 3 0 0 0	63	49	2	2	2	2	40.1771	2
11-4.3	1 5 6	3	49 3 0 0 0	63	49	2	3	3	3	40.1778	3
11-4.4	1 6 4	4	49 3 0 0 0	63	49	2	4	4	4	40.1783	4
11-4.5	1 6 5	5	49 3 0 0 0	63	49	2	5	5	5	40.1784	5
11-4.6	1 6 6	6	49 3 0 0 0	63	49	2	6	6	6	40.1784	6
11-4.7	1 7 4	7	49 3 0 0 0	63	49	2	7	7	7	40.1789	7
11-4.8	2 0 12	8	43 6 0 0 0	60	43	2	8	8	8	40.1809	8

k = 11, Designs sorted based on minimizing Lmax

Design	wlp(w ₁ ,...)	wlp rank	alp	df	C2FI	Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank
11-4.1	0	6	6	1	55	0	0	0	0	1	40.1723
11-4.2	1	4	6	2	49	3	0	0	0	2	40.1771
11-4.3	1	5	6	3	49	3	0	0	0	3	40.1778
11-4.4	1	6	4	4	49	3	0	0	0	4	40.1783
11-4.5	1	6	5	5	49	3	0	0	0	5	40.1784
11-4.6	1	6	6	6	49	3	0	0	0	6	40.1784
11-4.7	1	7	4	7	49	3	0	0	0	7	40.1789
11-4.8	2	0	12	8	43	6	0	0	0	8	40.1809

k = 11, Design generators

Design	Design Generators				
11-4.1	15	51	85	120	
11-4.2	7	57	90	108	
11-4.3	7	27	45	120	
11-4.4	7	27	45	121	
11-4.5	7	27	45	85	
11-4.6	7	27	45	78	
11-4.7	7	27	61	120	
11-4.8	7	59	93	112	
11-4.9a	7	26	45	121	
11-4.9b	7	27	45	112	
11-4.9c	7	51	93	112	
11-4.12	7	25	43	120	
11-4.13a	7	27	60	121	
11-4.13b	7	27	43	121	
11-4.15	7	27	58	121	
11-4.16	7	27	43	120	
11-4.17	7	51	85	112	
11-4.18	7	26	44	121	
11-4.19	7	11	61	94	
11-4.20	7	25	42	116	

k = 12, Designs sorted based on word length pattern

Design	wlp(w ₁ ,...)		wlp rank	alp		df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2
						rank	rank	rank	rank	rank	rank	rank	rank
12-5.1	1	8	12	1	60	3	0	0	0	0	0	36.1623	1
12-5.2	1	10	10	2	60	3	0	0	0	0	0	36.1633	2
12-5.3	1	10	11	3	60	3	0	0	0	0	0	36.1634	3
12-5.4	2	7	12	4	54	6	0	0	0	0	0	36.1672	4
12-5.5	2	8	10	5	54	6	0	0	0	0	0	36.1676	5
12-5.6	2	8	12	6	54	6	0	0	0	0	0	36.1677	6
12-5.7	2	9	9	7	54	6	0	0	0	0	0	36.1682	7
12-5.8a	2	10	8	8	54	6	0	0	0	0	0	36.1687	8
12-5.8b	2	10	8	8	54	6	0	0	0	0	0	36.1687	8
12-5.10	2	10	10	10	54	6	0	0	0	0	0	36.1688	10
12-5.11	2	11	8	11	54	6	0	0	0	0	0	36.1693	12
12-5.12	2	12	8	12	54	6	0	0	0	0	0	36.1699	13
12-5.13	3	0	24	13	48	9	0	0	0	0	0	36.1691	11
12-5.14	3	6	10	14	48	9	0	0	0	0	0	36.1719	14
12-5.15	3	6	11	15	51	6	1	0	0	0	0	36.1720	15
12-5.16	3	7	10	16	48	9	0	0	0	0	0	36.1725	16
12-5.17	3	8	7	17	51	6	1	0	0	0	0	36.1729	17
12-5.18a	3	8	7	17	48	9	0	0	0	0	0	36.1729	17
12-5.18b	3	8	7	17	48	9	0	0	0	0	0	36.1729	17
12-5.18c	3	8	7	17	48	9	0	0	0	0	0	36.1729	17

k = 12, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
12-5.1	1 8 12	1	60 3 0 0 0 0 0	75	60	2	1	1	1	36.1623	1
12-5.2	1 10 10	2	60 3 0 0 0 0 0	75	60	2	2	2	2	36.1633	2
12-5.3	1 10 11	3	60 3 0 0 0 0 0	75	60	2	3	3	3	36.1634	3
12-5.4	2 7 12	4	54 6 0 0 0 0 0	72	54	2	4	4	4	36.1672	4
12-5.5	2 8 10	5	54 6 0 0 0 0 0	72	54	2	5	5	5	36.1676	5
12-5.6	2 8 12	6	54 6 0 0 0 0 0	72	54	2	6	6	6	36.1677	6
12-5.7	2 9 9	7	54 6 0 0 0 0 0	72	54	2	7	7	7	36.1682	7

k = 12, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
12-5.1	1 8 12	1	60 3 0 0 0 0 0	75	60	2	1	1	1	36.1623	1
12-5.2	1 10 10	2	60 3 0 0 0 0 0	75	60	2	2	2	2	36.1633	2
12-5.3	1 10 11	3	60 3 0 0 0 0 0	75	60	2	3	3	3	36.1634	3
12-5.4	2 7 12	4	54 6 0 0 0 0 0	72	54	2	4	4	4	36.1672	4
12-5.5	2 8 10	5	54 6 0 0 0 0 0	72	54	2	5	5	5	36.1676	5
12-5.6	2 8 12	6	54 6 0 0 0 0 0	72	54	2	6	6	6	36.1677	6
12-5.7	2 9 9	7	54 6 0 0 0 0 0	72	54	2	7	7	7	36.1682	7

k = 12, Designs sorted based on minimizing Lmax

Design	wlp(w ₄ ,...)		wlp		alp		df		C2FI		Lmax		C2FI		Lmax		CD2*		CD2	
				rank						rank		rank		rank		rank			rank	
12-5.1	1	8	12	1	60	3	0	0	0	0	75	60	2	1	1	36.1623			1	
12-5.2	1	10	10	2	60	3	0	0	0	0	75	60	2	2	2	36.1633			2	
12-5.3	1	10	11	3	60	3	0	0	0	0	75	60	2	3	3	36.1634			3	
12-5.4	2	7	12	4	54	6	0	0	0	0	72	54	2	4	4	36.1672			4	
12-5.5	2	8	10	5	54	6	0	0	0	0	72	54	2	5	5	36.1676			5	
12-5.6	2	8	12	6	54	6	0	0	0	0	72	54	2	6	6	36.1677			6	
12-5.7	2	9	9	7	54	6	0	0	0	0	72	54	2	7	7	36.1682			7	

k = 12, Design generators

Design	Design Generators				
12-5.1	7	57	90	108	119
12-5.2	7	27	45	78	121
12-5.3	7	27	45	86	120
12-5.4	7	27	45	78	120
12-5.5	7	27	45	94	112
12-5.6	7	27	43	77	120
12-5.7	7	25	43	77	120
12-5.8a	7	25	43	85	120
12-5.8b	7	27	43	85	120
12-5.10	7	27	43	53	120
12-5.11	7	27	45	62	120
12-5.12	7	27	43	61	120
12-5.13	7	59	93	110	112
12-5.14	7	26	44	78	121
12-5.15	7	11	53	86	120
12-5.16	7	25	42	77	120
12-5.17	7	29	46	121	122
12-5.18a	7	27	45	112	121
12-5.18b	7	26	45	77	121
12-5.18c	7	26	45	86	121

k = 13, Designs sorted based on word length pattern

Design	wlp(w ₁ ,...)	wlp rank	alp	df	C2FI	Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank
13-6.1	2 16 18	1	66 6 0 0 0 0	85	66	2	1	1	1	32.5558	1
13-6.2	2 16 20	2	66 6 0 0 0 0	85	66	2	2	2	2	32.5559	2
13-6.3	3 12 24	3	60 9 0 0 0 0	82	60	2	4	4	3	32.5589	3
13-6.4	3 14 17	4	60 9 0 0 0 0	82	60	2	5	5	4	32.5596	4
13-6.5	3 14 18	5	60 9 0 0 0 0	82	60	2	6	6	5	32.5597	5
13-6.6	3 15 15	6	63 6 1 0 0 0	83	63	3	3	3	45	32.5600	6
13-6.7a	3 15 17	7	60 9 0 0 0 0	82	60	2	7	7	6	32.5601	7
13-6.7b	3 15 17	7	60 9 0 0 0 0	82	60	2	7	7	6	32.5601	7
13-6.9	3 16 15	9	60 9 0 0 0 0	82	60	2	9	9	8	32.5606	9
13-6.10	3 16 16	10	60 9 0 0 0 0	82	60	2	10	10	9	32.5606	10
13-6.11	3 17 15	11	60 9 0 0 0 0	82	60	2	11	11	10	32.5611	11
13-6.12	4 10 22	12	57 9 1 0 0 0	80	57	3	12	12	46	32.5627	12
13-6.13	4 12 16	13	54 12 0 0 0 0	79	54	2	29	30	11	32.5634	13
13-6.14	4 12 17	14	57 9 1 0 0 0	80	57	3	13	13	47	32.5635	14
13-6.15	4 12 18	15	57 9 1 0 0 0	80	57	3	14	14	48	32.5635	15
13-6.16	4 12 22	16	57 9 1 0 0 0	80	57	3	15	15	49	32.5637	16
13-6.17	4 13 16	17	54 12 0 0 0 0	79	54	2	30	31	12	32.5640	17
13-6.18	4 14 14	18	57 9 1 0 0 0	80	57	3	16	16	50	32.5644	19
13-6.19	4 14 14	18	54 12 0 0 0 0	79	54	2	31	32	13	32.5644	18
13-6.20	4 14 15	20	54 12 0 0 0 0	79	54	2	32	33	14	32.5644	20

k = 13, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
13-6.1	2 16 18	1	66 6 0 0 0 0	85	66	2	1	1	1	32.5558	1
13-6.2	2 16 20	2	66 6 0 0 0 0	85	66	2	2	2	2	32.5559	2
13-6.6	3 15 15	6	63 6 1 0 0 0	83	63	3	3	3	45	32.5600	6
13-6.3	3 12 24	3	60 9 0 0 0 0	82	60	2	4	4	3	32.5589	3
13-6.4	3 14 17	4	60 9 0 0 0 0	82	60	2	5	5	4	32.5596	4
13-6.5	3 14 18	5	60 9 0 0 0 0	82	60	2	6	6	5	32.5597	5
13-6.7b	3 15 17	7	60 9 0 0 0 0	82	60	2	7	7	6	32.5601	7
13-6.7a	3 15 17	7	60 9 0 0 0 0	82	60	2	7	7	6	32.5601	7
13-6.9	3 16 15	9	60 9 0 0 0 0	82	60	2	9	9	8	32.5606	9
13-6.10	3 16 16	10	60 9 0 0 0 0	82	60	2	10	10	9	32.5606	10

k = 13, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
13-6.1	2 16 18	1	66 6 0 0 0 0	85	66	2	1	1	1	32.5558	1
13-6.2	2 16 20	2	66 6 0 0 0 0	85	66	2	2	2	2	32.5559	2
13-6.6	3 15 15	6	63 6 1 0 0 0	83	63	3	3	3	45	32.5600	6
13-6.3	3 12 24	3	60 9 0 0 0 0	82	60	2	4	4	3	32.5589	3
13-6.4	3 14 17	4	60 9 0 0 0 0	82	60	2	5	5	4	32.5596	4
13-6.5	3 14 18	5	60 9 0 0 0 0	82	60	2	6	6	5	32.5597	5
13-6.7a	3 15 17	7	60 9 0 0 0 0	82	60	2	7	7	6	32.5601	7
13-6.7b	3 15 17	7	60 9 0 0 0 0	82	60	2	7	7	6	32.5601	7
13-6.9	3 16 15	9	60 9 0 0 0 0	82	60	2	9	9	8	32.5606	9
13-6.10	3 16 16	10	60 9 0 0 0 0	82	60	2	10	10	9	32.5606	10

k = 13, Designs sorted based on minimizing Lmax

Design	wlp(w ₁ ,...)	wlp rank	alp	df	C2FI	Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank
13-6.1	2 16 18	1	66 6 0 0 0 0 0	85	66	2	1	1	1	32.5558	1
13-6.2	2 16 20	2	66 6 0 0 0 0 0	85	66	2	2	2	2	32.5559	2
13-6.3	3 12 24	3	60 9 0 0 0 0 0	82	60	2	4	4	3	32.5589	3
13-6.4	3 14 17	4	60 9 0 0 0 0 0	82	60	2	5	5	4	32.5596	4
13-6.5	3 14 18	5	60 9 0 0 0 0 0	82	60	2	6	6	5	32.5597	5
13-6.7b	3 15 17	7	60 9 0 0 0 0 0	82	60	2	7	7	6	32.5601	7
13-6.7a	3 15 17	7	60 9 0 0 0 0 0	82	60	2	7	7	6	32.5601	7
13-6.9	3 16 15	9	60 9 0 0 0 0 0	82	60	2	9	9	8	32.5606	9
13-6.10	3 16 16	10	60 9 0 0 0 0 0	82	60	2	10	10	9	32.5606	10
13-6.11	3 17 15	11	60 9 0 0 0 0 0	82	60	2	11	11	10	32.5611	11

k = 13, Design generators

Design	Design Generators					
13-6.1	7	27	43	85	102	120
13-6.2	7	27	43	53	78	120
13-6.3	7	27	43	77	117	120
13-6.4	7	25	43	77	118	120
13-6.5	7	25	42	77	118	120
13-6.6	7	27	45	78	121	122
13-6.7a	7	25	42	53	78	120
13-6.7b	7	25	43	75	117	120
13-6.9	7	25	43	77	110	120
13-6.10	7	27	43	61	77	120
13-6.11	7	25	43	75	109	120
13-6.12	7	11	53	85	110	120
13-6.13	7	26	44	78	119	121
13-6.14	7	11	49	85	110	120
13-6.15	7	11	53	85	102	120
13-6.16	7	27	29	46	78	120
13-6.17	7	25	42	53	86	120
13-6.18	7	27	43	85	110	120
13-6.19	7	25	43	53	95	120
13-6.20	7	25	42	77	94	120

k = 14, Designs sorted based on word length pattern

Design	wlp(w ₁ ...)	wlp rank	alp	df	C2FI	Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank
14-7.1	3 24 36	1	73 9 0 0 0 0 0	96	73	2	1	1	1	29.3097	1
14-7.2	4 24 30	2	67 12 0 0 0 0 0	93	67	2	2	3	2	29.3138	2
14-7.3	5 22 30	3	64 12 1 0 0 0 0	91	64	3	3	4	33	29.3173	3
14-7.4	5 22 30	3	61 15 0 0 0 0 0	90	61	2	10	12	3	29.3173	3
14-7.5	5 23 27	5	64 12 1 0 0 0 0	91	64	3	4	5	34	29.3177	5
14-7.6a	5 23 27	5	61 15 0 0 0 0 0	90	61	2	11	13	4	29.3177	5
14-7.6b	5 23 27	5	61 15 0 0 0 0 0	90	61	2	11	13	4	29.3177	5
14-7.8a	5 24 26	8	64 12 1 0 0 0 0	91	64	3	5	6	35	29.3181	9
14-7.8b	5 24 26	8	64 12 1 0 0 0 0	91	64	3	5	6	35	29.3181	9
14-7.10a	5 24 26	8	61 15 0 0 0 0 0	90	61	2	13	15	6	29.3181	8
14-7.10b	5 24 26	8	61 15 0 0 0 0 0	90	61	2	13	15	6	29.3181	8
14-7.12	5 24 28	12	64 12 1 0 0 0 0	91	64	3	7	8	37	29.3182	12
14-7.13	5 26 26	13	64 12 1 0 0 0 0	91	64	3	8	9	38	29.3190	13
14-7.14	6 17 40	14	61 12 2 0 0 0 0	89	61	3	15	17	39	29.3198	14
14-7.15	6 20 28	15	61 12 2 0 0 0 0	89	61	3	16	18	40	29.3207	15
14-7.16	6 20 28	15	58 15 1 0 0 0 0	88	58	3	31	33	41	29.3207	15
14-7.17a	6 20 28	15	55 18 0 0 0 0 0	87	55	2	51	69	8	29.3207	15
14-7.17b	6 20 28	15	55 18 0 0 0 0 0	87	55	2	51	69	8	29.3207	15
14-7.19a	6 20 30	19	55 18 0 0 0 0 0	87	55	2	53	71	10	29.3208	19
14-7.19b	6 20 30	19	55 18 0 0 0 0 0	87	55	2	53	71	10	29.3208	19

k = 14, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	lmax	df rank	C2FI rank	lmax rank	CD2*	CD2 rank
14-7.1	3 24 36	1	73 9 0 0 0 0 0 0	96	73	2	1	1	1	29.3097	1
14-7.2	4 24 30	2	67 12 0 0 0 0 0 0	93	67	2	2	3	2	29.3138	2
14-7.3	5 22 30	3	64 12 1 0 0 0 0 0	91	64	3	3	4	33	29.3173	3
14-7.5	5 23 27	5	64 12 1 0 0 0 0 0	91	64	3	4	5	34	29.3177	5
14-7.8b	5 24 26	8	64 12 1 0 0 0 0 0	91	64	3	5	6	35	29.3181	9
14-7.8a	5 24 26	8	64 12 1 0 0 0 0 0	91	64	3	5	6	35	29.3181	9
14-7.12	5 24 28	12	64 12 1 0 0 0 0 0	91	64	3	7	8	37	29.3182	12
14-7.13	5 26 26	13	64 12 1 0 0 0 0 0	91	64	3	8	9	38	29.3190	13
14-7.94	7 21 21	94	70 0 7 0 0 0 0 0	91	70	3	9	2	98	29.3253	93
14-7.4	5 22 30	3	61 15 0 0 0 0 0 0	90	61	2	10	12	3	29.3173	3

k = 14, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	lmax	df rank	C2FI rank	lmax rank	CD2*	CD2 rank
14-7.1	3 24 36	1	73 9 0 0 0 0 0 0	96	73	2	1	1	1	29.3097	1
14-7.94	7 21 21	94	70 0 7 0 0 0 0 0	91	70	3	9	2	98	29.3253	93
14-7.2	4 24 30	2	67 12 0 0 0 0 0 0	93	67	2	2	3	2	29.3138	2
14-7.3	5 22 30	3	64 12 1 0 0 0 0 0	91	64	3	3	4	33	29.3173	3
14-7.5	5 23 27	5	64 12 1 0 0 0 0 0	91	64	3	4	5	34	29.3177	5
14-7.8a	5 24 26	8	64 12 1 0 0 0 0 0	91	64	3	5	6	35	29.3181	9
14-7.8b	5 24 26	8	64 12 1 0 0 0 0 0	91	64	3	5	6	35	29.3181	9
14-7.12	5 24 28	12	64 12 1 0 0 0 0 0	91	64	3	7	8	37	29.3182	12
14-7.13	5 26 26	13	64 12 1 0 0 0 0 0	91	64	3	8	9	38	29.3190	13
14-7.216	8 21 18	216	64 3 7 0 0 0 0 0	88	64	3	50	10	204	29.3296	216

k = 14, Designs sorted based on minimizing Lmax

Design	wlp(w ₁ ,...)	wlp rank	alp	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
14-7.1	3 24 36	1	73	9	0	0	0	0	0	29.3097	1
14-7.2	4 24 30	2	67	12	0	0	0	0	0	29.3138	2
14-7.4	5 22 30	3	61	15	0	0	0	0	0	29.3173	3
14-7.6b	5 23 27	5	61	15	0	0	0	0	0	29.3177	5
14-7.6a	5 23 27	5	61	15	0	0	0	0	0	29.3177	5
14-7.10b	5 24 26	8	61	15	0	0	0	0	0	29.3181	8
14-7.10a	5 24 26	8	61	15	0	0	0	0	0	29.3181	8

k = 14, Design generators

Design	Design Generators									
14-7.1	7	27	43	53	78	118	120			
14-7.2	7	25	42	53	78	118	120			
14-7.3	7	11	29	53	94	102	120			
14-7.4	7	25	42	53	78	83	120			
14-7.5	7	11	29	49	82	102	120			
14-7.6a	7	25	42	53	75	87	120			
14-7.6b	7	25	42	53	75	118	120			
14-7.8a	7	11	29	46	83	102	120			
14-7.8b	7	11	29	49	94	102	120			
14-7.10a	7	25	42	53	78	93	120			
14-7.10b	7	25	42	60	77	118	120			
14-7.12	7	11	29	45	78	118	120			
14-7.13	7	11	29	45	51	78	120			
14-7.14	7	27	29	46	78	118	120			
14-7.15	7	11	25	53	85	110	120			
14-7.16	7	11	29	53	86	102	120			
14-7.17a	7	25	42	53	76	86	120			
14-7.17b	7	25	42	53	86	102	120			
14-7.19a	7	25	42	53	83	92	120			
14-7.19b	7	25	42	61	78	118	120			
14-7.94	7	27	45	78	121	122	124			
14-7.216	7	27	43	85	94	101	120			

k = 15, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank
15-8.1	7 32 52	1	63 21	0	0	0	0	0	26.3993	1
15-8.2	7 34 46	2	63 21	0	0	0	0	0	26.3999	2
15-8.3	7 38 44	3	69 15	2	0	0	0	0	26.4015	3
15-8.4	8 31 50	4	57 24	0	0	0	0	0	26.4028	4
15-8.5	8 32 44	5	57 24	0	0	0	0	0	26.4030	5
15-8.6	8 32 49	6	63 18	2	0	0	0	0	26.4032	6
15-8.7	8 32 49	6	57 24	0	0	0	0	0	26.4032	7
15-8.8	8 33 44	8	60 21	1	0	0	0	0	26.4034	8
15-8.9	8 33 44	9	66 15	3	0	0	0	0	26.4034	8
15-8.10	8 33 44	9	60 21	1	0	0	0	0	26.4034	8
15-8.11	8 33 44	11	60 21	1	0	0	0	0	26.4034	8
15-8.12	8 33 44	11	57 24	0	0	0	0	0	26.4034	8
15-8.13	8 34 42	13	63 18	2	0	0	0	0	26.4037	13
15-8.14a	8 34 42	13	60 21	1	0	0	0	0	26.4037	13
15-8.14b	8 34 42	13	60 21	1	0	0	0	0	26.4037	13
15-8.14c	8 34 42	13	60 21	1	0	0	0	0	26.4037	13
15-8.17	8 34 43	17	60 21	1	0	0	0	0	26.4038	17
15-8.18	8 34 43	18	66 15	3	0	0	0	0	26.4038	17
15-8.19	8 34 46	19	60 21	1	0	0	0	0	26.4039	19
15-8.20	8 35 42	20	66 15	3	0	0	0	0	26.4041	20

k = 15, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	lmax	df	C2FI	lmax	CD2*	CD2
						rank	rank	rank	rank	rank	rank	rank	rank
15-8.3	7 38 44	3	69 15	2	0 0 0	101	69	3	1	3	13	26.4015	3
15-8.1	7 32 52	1	63 21	0	0 0 0	99	63	2	2	11	1	26.3993	1
15-8.2	7 34 46	2	63 21	0	0 0 0	99	63	2	3	12	2	26.3999	2
15-8.9	8 33 44	9	66 15	3	0 0 0	99	66	3	4	4	16	26.4034	8
15-8.18	8 34 43	18	66 15	3	0 0 0	99	66	3	5	5	24	26.4038	17
15-8.20	8 35 42	20	66 15	3	0 0 0	99	66	3	6	6	26	26.4041	20
15-8.1221	14 28 28	1221	77 0	0	7 0 0	99	77	4	7	1	1366	26.4245	1226
15-8.6	8 32 49	6	63 18	2	0 0 0	98	63	3	8	13	14	26.4032	6
15-8.13	8 34 42	13	63 18	2	0 0 0	98	63	3	9	14	19	26.4037	13
15-8.22b	8 36 41	22	63 18	2	0 0 0	98	63	3	10	16	28	26.4045	22

k = 15, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	lmax	df	C2FI	lmax	CD2*	CD2
						rank	rank	rank	rank	rank	rank	rank	rank
15-8.1221	14 28 28	1221	77 0	0	7 0 0	99	77	4	7	1	1366	26.4245	1226
15-8.1578	15 28 24	1578	71 3	0	7 0 0	96	71	4	57	2	1615	26.4284	1593
15-8.3	7 38 44	3	69 15	2	0 0 0	101	69	3	1	3	13	26.4015	3
15-8.9	8 33 44	9	66 15	3	0 0 0	99	66	3	4	4	16	26.4034	8
15-8.18	8 34 43	18	66 15	3	0 0 0	99	66	3	5	5	24	26.4038	17
15-8.20	8 35 42	20	66 15	3	0 0 0	99	66	3	6	6	26	26.4041	20
15-8.152	10 32 37	152	66 9	7	0 0 0	97	66	3	27	7	148	26.4106	153
15-8.303	11 30 36	303	65 9	6	1 0 0	96	65	4	53	8	933	26.4137	303
15-8.344	11 31 34	344	65 9	6	1 0 0	96	65	4	55	9	940	26.4141	352
15-8.358	11 32 34	358	65 9	6	1 0 0	96	65	4	56	10	944	26.4145	363

k = 15, Designs sorted based on minimizing Lmax

Design	wlp(w ₁ ,...)	wlp rank	alp			df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
15-8.1	7 32 52	1	63	21	0	0	0	0	99	63	2	26.3993	1
15-8.2	7 34 46	2	63	21	0	0	0	0	99	63	2	26.3999	2
15-8.4	8 31 50	4	57	24	0	0	0	0	96	57	2	26.4028	4
15-8.5	8 32 44	5	57	24	0	0	0	0	96	57	2	26.4030	5
15-8.7	8 32 49	6	57	24	0	0	0	0	96	57	2	26.4032	7
15-8.12	8 33 44	11	57	24	0	0	0	0	96	57	2	26.4034	8
15-8.26	9 28 48	26	51	27	0	0	0	0	93	51	2	26.4054	25
15-8.31	9 30 46	27	51	27	0	0	0	0	93	51	2	26.4062	30
15-8.45	9 32 42	41	51	27	0	0	0	0	93	51	2	26.4069	41
15-8.214	11 20 60	214	39	33	0	0	0	0	87	39	2	26.4104	149

k = 15, Design generators

Design	Design Generators										
15-8.1	7	25	42	53	78	83	111	120			
15-8.2	7	25	42	53	75	87	116	120			
15-8.3	7	11	29	45	51	78	118	120			
15-8.4	7	25	42	53	62	78	83	120			
15-8.5	7	25	42	53	75	87	108	120			
15-8.6	7	11	29	46	53	83	107	120			
15-8.7	7	25	42	53	62	78	92	120			
15-8.8	7	11	29	45	62	81	98	120			
15-8.9	7	11	25	45	50	86	110	120			
15-8.10	7	11	29	46	49	82	102	120			
15-8.11	7	11	29	46	49	82	109	120			
15-8.12	7	25	42	52	63	77	91	120			
15-8.13	7	11	25	45	55	86	100	120			
15-8.14a	7	11	29	45	62	81	99	120			
15-8.14b	7	11	29	46	49	83	102	120			
15-8.14c	7	11	29	46	49	83	109	120			
15-8.17	7	11	29	45	62	81	106	120			
15-8.18	7	11	25	42	53	78	118	120			
15-8.19	7	11	29	46	53	83	94	120			
15-8.20	7	11	25	45	49	86	110	120			
15-8.22b	7	11	29	45	51	78	86	120			
15-8.26	7	25	42	52	77	86	107	120			
15-8.31	7	25	42	52	63	77	86	120			
15-8.45	7	25	42	52	63	77	107	120			
15-8.152	7	11	13	30	49	82	101	120			
15-8.214	7	25	42	52	77	86	119	120			
15-8.303	7	11	19	25	45	77	118	120			
15-8.344	7	11	19	25	45	86	100	120			
15-8.358	7	11	19	25	45	77	110	120			
15-8.1221	7	27	45	78	121	122	124	127			
15-8.1578	7	27	43	85	94	101	110	120			

k = 16, Designs sorted based on word length pattern

Design	wlp(w ₁ ,...)		wlp rank	alp		df	C2FI	Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank
16-9.1	10	48	72	1	60	30	0	0	0	0	0	23.7778	1
16-9.2	11	44	82	2	54	33	0	0	0	0	0	23.7802	2
16-9.3	11	47	72	3	57	30	1	0	0	0	0	23.7810	3
16-9.4	11	48	70	4	57	30	1	0	0	0	0	23.7813	4
16-9.5	11	50	66	5	60	27	2	0	0	0	0	23.7819	5
16-9.6	11	50	68	6	60	27	2	0	0	0	0	23.7819	6
16-9.7	11	52	66	7	60	27	2	0	0	0	0	23.7826	8
16-9.8	11	56	66	8	66	21	4	0	0	0	0	23.7842	16
16-9.9	12	40	80	9	48	36	0	0	0	0	0	23.7822	7
16-9.10	12	46	68	10	60	24	4	0	0	0	0	23.7840	9
16-9.11	12	46	68	10	57	27	3	0	0	0	0	23.7840	9
16-9.12a	12	46	68	10	54	30	2	0	0	0	0	23.7840	9
16-9.12b	12	46	68	10	54	30	2	0	0	0	0	23.7840	9
16-9.14a	12	46	69	14	54	30	2	0	0	0	0	23.7840	13
16-9.14b	12	46	69	14	54	30	2	0	0	0	0	23.7840	13
16-9.16	12	46	69	14	51	33	1	0	0	0	0	23.7840	13
16-9.17a	12	47	66	17	60	24	4	0	0	0	0	23.7840	13
16-9.17b	12	47	66	17	60	24	4	0	0	0	0	23.7843	17
16-9.19a	12	47	66	17	57	27	3	0	0	0	0	23.7843	17
16-9.19b	12	47	66	17	57	27	3	0	0	0	0	23.7843	17

k = 16, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	lmax	df	C2FI	lmax	CD2*	CD2 rank			
									rank	rank	rank					
16-9.8	11 56 66	8	66	21	4	0	0	0	107	66	3	1	6	11	23.7842	16
16-9.1	10 48 72	1	60	30	0	0	0	0	106	60	2	2	24	1	23.7778	1
16-9.5	11 50 66	5	60	27	2	0	0	0	105	60	3	3	25	8	23.7819	5
16-9.6	11 50 68	6	60	27	2	0	0	0	105	60	3	4	26	9	23.7819	6
16-9.7	11 52 66	7	60	27	2	0	0	0	105	60	3	5	27	10	23.7826	8
16-9.35	12 50 63	35	63	21	5	0	0	0	105	63	3	6	11	37	23.7854	37
16-9.39	12 52 63	39	63	21	5	0	0	0	105	63	3	7	12	41	23.7861	40
16-9.80	13 46 66	80	65	18	5	1	0	0	105	65	4	8	7	803	23.7875	80
16-9.90	13 47 64	90	65	18	5	1	0	0	105	65	4	9	8	806	23.7878	91

k = 16, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	lmax	df rank	C2FI rank	lmax rank	CD2*	CD2 rank				
16-9.1413	17 43 56	1413	69	6	9	3	0	0	0	103	69	4	46	1	1551	23.8004	1446
16-9.2469	19 40 50	2469	69	11	0	6	1	0	0	103	69	5	47	2	4905	23.8062	2578
16-9.2499	19 41 48	2499	69	11	0	6	1	0	0	103	69	5	48	3	4911	23.8065	2647
16-9.2531	19 42 48	2531	69	11	0	6	1	0	0	103	69	5	49	4	4917	23.8069	2696
16-9.225	14 46 61	225	67	15	5	2	0	0	0	105	67	4	11	5	842	23.7909	232
16-9.8	11 56 66	8	66	21	4	0	0	0	0	107	66	3	1	6	11	23.7842	16
16-9.80	13 46 66	80	65	18	5	1	0	0	0	105	65	4	8	7	803	23.7875	80
16-9.90	13 47 64	90	65	18	5	1	0	0	0	105	65	4	9	8	806	23.7878	91

k = 16, Designs sorted based on minimizing Lmax

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
16-9.1	10 48 72	1	60	30	0	0	0	0	0	0	0	23.7778	1
16-9.2	11 44 82	2	54	33	0	0	0	0	0	0	0	23.7802	2
16-9.9	12 40 80	9	48	36	0	0	0	0	0	0	0	23.7822	7
16-9.287	15 30 100	287	30	45	0	0	0	0	0	0	0	23.7897	142
16-9.2604	20 0 160	2604	0	60	0	0	0	0	0	0	0	23.7982	1042
16-9.3	11 47 72	3	57	30	1	0	0	0	0	0	0	23.7810	3
16-9.4	11 48 70	4	57	30	1	0	0	0	0	0	0	23.7813	4
16-9.5	11 50 66	5	60	27	2	0	0	0	0	0	0	23.7819	5
16-9.6	11 50 68	6	60	27	2	0	0	0	0	0	0	23.7819	6
16-9.7	11 52 66	7	60	27	2	0	0	0	0	0	0	23.7826	8

k = 16, Design generators

Design	Design Generators									
16-9.1	7	120	25	42	53	75	87	108	118	
16-9.2	7	120	25	42	53	62	78	83	92	
16-9.3	7	120	11	29	45	51	78	81	111	
16-9.4	7	120	11	29	45	51	78	81	100	
16-9.5	7	120	11	29	45	51	78	81	107	
16-9.6	7	120	11	29	45	51	78	81	118	
16-9.7	7	120	11	29	45	51	62	78	81	
16-9.8	7	120	11	29	45	51	53	78	118	
16-9.9	7	120	25	42	52	77	86	107	119	
16-9.10	7	120	11	21	46	54	89	95	99	
16-9.11	7	120	11	21	41	51	78	86	100	
16-9.12a	7	120	11	29	45	49	78	86	106	
16-9.12b	7	120	11	21	45	62	86	91	97	
16-9.14a	7	120	11	29	45	53	78	81	98	
16-9.14b	7	120	11	25	45	51	78	90	101	
16-9.16	7	120	11	29	45	51	78	81	106	
16-9.17a	7	120	11	21	45	86	91	97	103	
16-9.17b	7	120	11	25	45	49	77	82	110	
16-9.19a	7	120	11	21	41	51	78	93	100	
16-9.19b	7	120	11	21	41	58	77	91	118	
16-9.35	7	120	11	25	45	50	60	86	110	
16-9.39	7	120	11	25	45	49	63	86	110	
16-9.80	7	120	11	19	29	41	44	94	102	
16-9.90	7	120	11	19	41	44	53	78	118	
16-9.225	7	120	11	19	25	41	53	78	118	
16-9.287	7	120	25	42	61	77	83	95	99	
16-9.1413	7	120	11	19	25	28	45	77	110	
16-9.2469	7	120	11	19	25	26	45	77	118	
16-9.2499	7	120	11	19	25	26	45	86	100	
16-9.2531	7	120	11	19	25	26	45	77	110	
16-9.2604	7	121	112	26	44	59	79	94	109	

k = 17, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)	wlp rank	alp				df	C2FI	lmax	df	C2FI	lmax	rank	C2FI	lmax	rank	CD2*	CD2 rank
17-10.1	15 60 130	1	46	45	0	0	0	0	0	0	108	46	2	53	1594	1	21.4231	1
17-10.2	15 66 110	2	52	39	2	0	0	0	0	0	110	52	3	6	390	3	21.4245	2
17-10.3	15 68 106	3	52	39	2	0	0	0	0	0	110	52	3	7	391	4	21.4251	3
17-10.4	15 72 102	4	58	33	4	0	0	0	0	0	112	58	3	1	62	5	21.4263	4
17-10.5	16 64 108	5	46	42	2	0	0	0	0	0	107	46	3	106	1594	6	21.4270	5
17-10.6	16 65 105	6	55	33	5	0	0	0	0	0	110	55	3	8	152	7	21.4273	6
17-10.7	16 65 105	6	52	36	4	0	0	0	0	0	109	52	3	22	392	8	21.4273	6
17-10.8	16 65 107	8	49	39	3	0	0	0	0	0	108	49	3	54	835	9	21.4273	8
17-10.9	16 66 102	9	55	33	5	0	0	0	0	0	110	55	3	9	153	10	21.4275	9
17-10.10	16 66 102	9	52	36	4	0	0	0	0	0	109	52	3	22	393	11	21.4275	9
17-10.11	16 67 101	11	55	33	5	0	0	0	0	0	110	55	3	10	154	12	21.4278	11
17-10.12	16 68 99	12	58	30	6	0	0	0	0	0	111	58	3	2	63	13	21.4281	12
17-10.13	16 68 100	13	55	33	5	0	0	0	0	0	110	55	3	11	155	14	21.4281	13
17-10.14	16 69 99	14	58	30	6	0	0	0	0	0	111	58	3	3	64	15	21.4284	14
17-10.15	16 69 99	14	55	33	5	0	0	0	0	0	110	55	3	12	156	16	21.4284	14
17-10.16	17 62 106	16	51	36	3	1	0	0	0	0	108	51	4	55	525	365	21.4295	16
17-10.17	17 62 108	17	49	36	5	0	0	0	0	0	107	49	3	107	836	17	21.4296	17
17-10.18	17 64 99	18	55	30	7	0	0	0	0	0	109	55	3	24	157	18	21.4300	18
17-10.19	17 64 100	19	51	36	3	1	0	0	0	0	108	51	4	56	526	366	21.4300	19
17-10.20	17 64 102	20	55	30	7	0	0	0	0	0	109	55	3	25	158	19	21.4301	20

k = 17, Designs sorted based on degrees of freedom used

Design	wlp(w ₁ ,...)	wlp rank	alp			df	C2FI	lmax	df	C2FI	lmax	CD2*	CD2
						rank	rank	rank	rank	rank	rank	rank	rank
17-10.4	15 72 102	4	58	33	4	0	0	0	112	58	3	21.4263	4
17-10.12	16 68 99	12	58	30	6	0	0	0	111	58	3	21.4281	12
17-10.14	16 69 99	14	58	30	6	0	0	0	111	58	3	21.4284	14
17-10.1042	21 62 92	1042	68	14	9	2	1	0	111	68	5	21.4419	1091
17-10.2453	23 60 86	2453	68	19	0	5	2	0	111	68	5	21.4475	2680
17-10.2	15 66 110	2	52	39	2	0	0	0	110	52	3	21.4245	2
17-10.3	15 68 106	3	52	39	2	0	0	0	110	52	3	21.4251	3
17-10.6	16 65 105	6	55	33	5	0	0	0	110	55	3	21.4273	6
17-10.9	16 66 102	9	55	33	5	0	0	0	110	55	3	21.4275	9
17-10.11	16 67 101	11	55	33	5	0	0	0	110	55	3	21.4278	11

k = 17, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₁ ,...)	wlp rank	alp			df	C2FI	lmax	df	C2FI	lmax	CD2*	CD2
						rank	rank	rank	rank	rank	rank	rank	rank
17-10.5924	27 56 82	5924	75	3	8	4	3	0	110	75	5	21.4589	6713
17-10.12633	39 44 86	12633	70	0	0	12	0	3	102	70	6	21.4938	13276
17-10.6792	28 55 77	6792	69	6	8	4	3	0	107	69	5	21.4617	7580
17-10.1042	21 62 92	1042	68	14	9	2	1	0	111	68	5	21.4419	1091
17-10.2453	23 60 86	2453	68	19	0	5	2	0	111	68	5	21.4475	2680
17-10.6795a	28 55 79	6795	66	6	6	10	0	0	105	66	4	21.4617	7626
17-10.6795b	28 55 79	6795	66	6	6	10	0	0	105	66	4	21.4617	7626
17-10.7585a	29 52 76	7585	66	6	9	4	3	0	105	66	5	21.4638	8165
17-10.7585b	29 52 76	7585	66	6	9	4	3	0	105	66	5	21.4638	8165

k = 17, Design generators

Design	Design Generators														
17-10.1	7	25	42	53	62	78	83	92	99	120					
17-10.2	7	11	29	45	51	78	81	100	118	120					
17-10.3	7	11	29	45	51	62	78	81	100	120					
17-10.4	7	11	25	45	51	62	78	84	90	120					
17-10.5	7	11	29	45	51	78	81	106	118	120					
17-10.6	7	11	21	45	62	86	91	97	103	120					
17-10.7	7	11	21	41	58	77	91	110	118	120					
17-10.8	7	11	25	45	51	62	78	84	101	120					
17-10.9	7	11	21	38	57	76	83	111	118	120					
17-10.10	7	11	25	45	51	77	87	98	118	120					
17-10.11	7	11	21	41	51	63	78	93	100	120					
17-10.12	7	11	21	45	59	78	86	97	103	120					
17-10.13	7	11	21	38	57	77	83	110	118	120					
17-10.14	7	11	21	45	54	83	93	97	103	120					
17-10.15	7	11	21	45	51	62	78	86	97	120					
17-10.16	7	11	19	41	53	74	85	110	118	120					
17-10.17	7	11	21	41	54	58	79	86	101	120					
17-10.18	7	11	21	38	57	73	83	108	118	120					
17-10.19	7	11	19	41	53	78	82	109	118	120					
17-10.20	7	11	21	41	50	63	78	84	101	120					
17-10.315	7	25	42	61	77	83	95	99	108	120					
17-10.1042	7	11	19	25	28	35	45	86	110	120					
17-10.2453	7	11	19	25	26	41	53	78	118	120					
17-10.5924	7	11	19	25	26	28	45	77	110	120					
17-10.6792	7	11	13	19	25	26	46	85	100	120					
17-10.6795a	7	11	13	19	21	25	46	78	98	120					
17-10.6795b	7	11	13	19	21	25	46	78	118	120					
17-10.7585a	7	11	13	19	25	26	53	85	110	120					
17-10.7585b	7	11	13	19	25	26	54	86	110	120					
17-10.7644a	7	11	13	19	25	26	46	78	118	120					
17-10.12633	7	11	19	25	26	28	31	45	78	120					

k = 18, Designs sorted based on word length pattern

Design	wlp(w ₁ ...)		wlp rank	alp		df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2
						rank	rank	rank	rank	rank	rank	rank	rank
18-11.1	20	80	200	1	33	60	0	0	0	0	0	19.3048	1
18-11.2	20	92	160	2	45	48	4	0	0	0	0	19.3074	2
18-11.3	21	95	148	3	54	36	9	0	0	0	0	19.3109	3
18-11.4	21	96	151	4	54	36	9	0	0	0	0	19.3112	4
18-11.5	22	86	162	5	42	45	7	0	0	0	0	19.3114	5
18-11.6	22	90	150	6	51	36	10	0	0	0	0	19.3123	6
18-11.7	22	90	150	6	48	39	9	0	0	0	0	19.3123	6
18-11.8	22	92	146	8	51	36	10	0	0	0	0	19.3128	10
18-11.9	22	92	146	8	50	39	7	1	0	0	0	19.3128	10
18-11.10a	22	92	146	10	53	36	8	1	0	0	0	19.3128	8
18-11.10b	22	92	146	10	53	36	8	1	0	0	0	19.3128	8
18-11.12a	22	92	148	12	50	39	7	1	0	0	0	19.3128	12
18-11.12b	22	92	148	12	50	39	7	1	0	0	0	19.3128	12
18-11.14	23	86	154	14	48	36	11	0	0	0	0	19.3141	14
18-11.15	23	86	154	14	44	42	7	1	0	0	0	19.3141	14
18-11.16	23	88	148	16	50	36	9	1	0	0	0	19.3145	16
18-11.17	23	88	148	16	48	36	11	0	0	0	0	19.3145	16
18-11.18	23	88	148	16	47	39	8	1	0	0	0	19.3145	16
18-11.19	23	88	148	16	43	45	4	2	0	0	0	19.3145	19
18-11.20	23	88	150	20	51	33	12	0	0	0	0	19.3146	20

k = 18, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp			df C2FI lmax			df rank	C2FI rank	lmax rank	CD2*	CD2 rank	
18-11.3	21 95 148	3	54	36	9	0	0	0	0	117	54	3	19.3109	3
18-11.4	21 96 151	4	54	36	9	0	0	0	0	117	54	3	19.3112	4
18-11.5146	32 80 132	5146	71	13	8	4	2	1	0	0	0	5 18580	19.3377	6205
18-11.14398	40 72 124	14398	81	3	0	12	0	3	0	0	0	1 20088	19.3583	16763
18-11.10b	22 92 146	10	53	36	8	1	0	0	0	0	0	155 70	19.3128	8
18-11.10a	22 92 146	10	53	36	8	1	0	0	0	0	0	155 70	19.3128	8
18-11.2	20 92 160	2	45	48	4	0	0	0	0	115	45	3	19.3074	2
18-11.6	22 90 150	6	51	36	10	0	0	0	0	115	51	3	19.3123	6
18-11.8	22 92 146	8	51	36	10	0	0	0	0	115	51	3	19.3128	10
18-11.9	22 92 146	8	50	39	7	1	0	0	0	115	50	4	19.3128	10

k = 18, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₁ ,...)			wlp			alp			df C2FI lmax			C2FI		lmax		CD2*		CD2		
													rank	rank	rank	rank	rank	rank	rank		
18-11.14398	40	72	124	14398	81	3	0	12	0	3	0	0	0	0	117	81	6	19.3583	16763	1	20088
18-11.15397a	41	71	120	15397	72	6	1	9	6	0	0	0	0	0	112	72	5	19.3608	17757	2	13773
18-11.15397b	41	71	120	15397	72	6	1	9	6	0	0	0	0	0	112	72	5	19.3608	17757	2	13773
18-11.16125	42	72	112	16125	72	6	1	12	0	3	0	0	0	0	112	72	6	19.3637	18906	4	20598
18-11.5146	32	80	132	5146	71	13	8	4	2	1	0	0	0	0	117	71	6	19.3377	6205	3	5
18-11.15386	41	70	120	15386	69	9	0	9	6	0	0	0	0	0	111	69	5	19.3605	17304	6	13769
18-11.23841a	56	56	140	23841	69	3	0	0	12	3	0	0	0	0	105	69	6	19.4004	24353	7	23076
18-11.23841b	56	56	140	23841	69	3	0	0	12	3	0	0	0	0	105	69	6	19.4004	24352	7	23076
18-11.5147	32	80	132	5146	66	14	6	9	1	0	0	0	0	0	114	66	5	19.3377	6205	9	7496
18-11.6397	33	79	128	6397	66	14	9	3	4	0	0	0	0	0	114	66	5	19.3402	7583	10	8258

k = 18, Designs sorted based on minimizing Lmax

Design	wlp(w ₁ ,...)		wlp rank	alp		df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank
18-11.1	20	80	200	1	33	60	0	0	0	0	0	0	1
18-11.2	20	92	160	2	45	48	4	0	0	0	0	19.3048	2
18-11.3	21	95	148	3	54	36	9	0	0	0	0	19.3074	3
18-11.4	21	96	151	4	54	36	9	0	0	0	0	19.3109	4
18-11.5	22	86	162	5	42	45	7	0	0	0	0	19.3112	5
18-11.6	22	90	150	6	51	36	10	0	0	0	0	19.3114	6
18-11.7	22	90	150	6	48	39	9	0	0	0	0	19.3123	7
18-11.8	22	92	146	8	51	36	10	0	0	0	0	19.3128	8
18-11.14	23	86	154	14	48	36	11	0	0	0	0	19.3141	9
18-11.17	23	88	148	16	48	36	11	0	0	0	0	19.3145	10
													16

k = 18, Design generators

Design	Design Generators																
18-11.1	7	25	42	53	62	78	83	92	99	111	120						
18-11.2	7	11	25	45	51	62	78	84	90	101	120						
18-11.3	7	11	21	45	51	62	78	86	97	103	120						
18-11.4	7	11	25	42	77	81	95	99	110	118	120						
18-11.5	7	11	21	41	54	58	79	86	92	99	120						
18-11.6	7	11	21	38	57	76	83	90	111	118	120						
18-11.7	7	11	21	38	57	76	83	90	101	118	120						
18-11.8	7	11	21	41	51	63	77	84	110	118	120						
18-11.9	7	11	19	41	53	63	78	82	99	118	120						
18-11.10a	7	11	19	29	41	53	74	84	111	118	120						
18-11.10b	7	11	19	38	57	60	77	85	91	101	120						
18-11.12a	7	11	19	41	53	63	78	82	95	99	120						
18-11.12b	7	11	19	41	53	63	78	82	95	100	120						
18-11.14	7	11	21	38	59	73	83	95	106	118	120						
18-11.15	7	11	19	29	41	53	74	85	110	118	120						
18-11.16	7	11	19	29	38	41	69	91	106	116	120						
18-11.17	7	11	21	25	38	58	78	84	101	107	120						
18-11.18	7	11	19	38	57	60	73	85	106	118	120						
18-11.19	7	11	19	29	41	53	73	86	102	106	120						
18-11.20	7	11	21	25	38	58	77	83	101	118	120						
18-11.5146	7	11	19	25	26	28	35	45	86	110	120						
18-11.5147	7	11	13	19	21	25	41	63	78	118	120						
18-11.6397	7	11	13	19	25	26	46	49	85	109	120						
18-11.14398	7	11	19	25	26	28	31	45	77	110	120						
18-11.15386	7	11	13	19	21	25	26	46	92	103	120						
18-11.15397a	7	11	13	19	21	25	26	46	78	100	120						
18-11.15397b	7	11	13	19	21	25	26	46	78	118	120						
18-11.16125	7	11	19	25	26	28	31	45	77	117	120						
18-11.23841a	7	11	13	19	21	25	26	28	46	78	120						
18-11.23841b	7	11	13	19	21	25	26	28	46	95	120						

k = 19, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)		wlp rank	alp		df C2FI		Lmax	df	C2FI rank	Lmax	CD2*	CD2 rank
19-12.1	27	120	235	1	36	54	9	0	0	0	0	17.4063	1
19-12.2	28	122	220	2	45	42	14	0	0	0	0	17.4091	2
19-12.3	30	110	240	3	32	51	11	1	0	0	0	17.4115	3
19-12.4	30	114	228	4	42	39	17	0	0	0	0	17.4123	5
19-12.5	30	116	220	5	40	45	11	2	0	0	0	17.4127	6
19-12.6	30	118	214	6	45	36	18	0	0	0	0	17.4131	7
19-12.7	30	118	214	7	47	36	16	1	0	0	0	17.4131	7
19-12.8	30	118	214	7	44	39	15	1	0	0	0	17.4131	7
19-12.9	30	118	216	9	44	39	15	1	0	0	0	17.4131	10
19-12.10	30	120	212	10	42	51	1	6	0	0	0	17.4135	11
19-12.11	30	121	208	11	47	36	16	1	0	0	0	17.4137	12
19-12.12	30	122	208	12	50	33	17	1	0	0	0	17.4140	13
19-12.13	30	122	208	12	46	45	5	5	0	0	0	17.4140	13
19-12.14	31	100	271	14	30	48	15	0	0	0	0	17.4121	4
19-12.15	31	116	210	15	43	39	14	2	0	0	0	17.4150	15
19-12.16	31	116	215	16	46	36	15	2	0	0	0	17.4151	16
19-12.17a	31	116	215	16	40	42	13	2	0	0	0	17.4151	16
19-12.17b	31	116	215	16	40	42	13	2	0	0	0	17.4151	16
19-12.19	31	116	219	19	50	30	19	1	0	0	0	17.4152	19
19-12.20a	31	117	210	20	46	36	15	2	0	0	0	17.4153	20
19-12.20b	31	117	210	20	46	36	15	2	0	0	0	17.4153	20

k = 19, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2
							rank	rank	rank	rank	rank	rank	rank
19-12.12482	46 102 192 12482	74 15 0 12 0 2 1 0 0	0	0	0	123	74	7	1	3	35208	17.4498	16695
19-12.6923	42 106 200 6923	70 8 17 2 4 1 0 0 0	0	0	0	121	70	6	2	7	22319	17.4407	9180
19-12.2	28 122 220 2	45 42 14 0 0 0 0 0 0	0	0	0	120	45	3	3	681	2	17.4091	2
19-12.13	30 122 208 12	46 45 5 0 0 0 0 0 0	0	0	0	120	46	4	4	517	15	17.4140	13
19-12.12	30 122 208 12	50 33 17 1 0 0 0 0 0	0	0	0	120	50	4	5	170	16	17.4140	13
19-12.161	33 117 198 161	53 32 11 4 1 0 0 0 0	0	0	0	120	53	5	6	91	2587	17.4202	164
19-12.3218	39 116 187 3218	59 26 9 4 1 2 0 0 0	0	0	0	120	59	6	7	39	21728	17.4353	5406
19-12.12483	46 102 192 12482	69 16 1 9 5 1 0 0 0	0	0	0	120	69	6	8	8	24025	17.4498	16695
19-12.14059	47 100 187 14059	68 18 0 12 0 2 1 0 0	0	0	0	120	68	7	9	11	35317	17.4518	18094
19-12.7	30 118 214 7	47 36 16 1 0 0 0 0 0	0	0	0	119	47	4	10	384	10	17.4131	7

k = 19, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2
							rank	rank	rank	rank	rank	rank	rank
19-12.26380a	58 90 184 26380	78 6 1 0 12 3 0 0 0	0	0	0	119	78	6	20	1	29308	17.4777	33773
19-12.26380b	58 90 184 26380	78 6 1 0 12 3 0 0 0	0	0	0	119	78	6	20	1	29308	17.4777	33773
19-12.12482	46 102 192 12482	74 15 0 12 0 2 1 0 0	0	0	0	123	74	7	1	3	35208	17.4498	16695
19-12.38700	78 70 224 38700	74 3 0 0 0 14 1 0 0	0	0	0	111	74	7	1911	4	38310	17.5257	38922
19-12.31264	62 86 164 31264	72 0 7 0 12 3 0 0 0	0	0	0	113	72	6	968	5	30857	17.4865	36481
19-12.31266	62 90 160 31266	72 0 7 0 12 3 0 0 0	0	0	0	113	72	6	969	6	30858	17.4875	36579
19-12.6923	42 106 200 6923	70 8 17 2 4 1 0 0 0	0	0	0	121	70	6	2	7	22319	17.4407	9180
19-12.12483	46 102 192 12482	69 16 1 9 5 1 0 0 0	0	0	0	120	69	6	8	8	24025	17.4498	16695
19-12.27425	59 86 182 27425	69 12 0 0 12 3 0 0 0	0	0	0	115	69	6	386	9	29630	17.4792	34647

k = 19, Design generators

Design	Design Generators																		
19-12.1	7	11	21	41	54	58	79	86	92	99	101	120							
19-12.2	7	11	21	38	57	76	83	90	101	111	118	120							
19-12.3	7	11	19	38	59	62	73	87	93	101	106	120							
19-12.4	7	11	21	38	59	73	83	95	101	106	118	120							
19-12.5	7	11	19	38	57	60	73	85	95	101	106	120							
19-12.6	7	11	21	38	57	73	83	95	101	107	118	120							
19-12.7	7	11	19	38	57	60	73	84	99	110	118	120							
19-12.8	7	11	19	38	57	60	73	85	99	110	118	120							
19-12.9	7	11	21	25	38	55	58	78	84	101	107	120							
19-12.10	7	11	19	30	41	52	61	74	87	101	111	120							
19-12.11	7	11	19	29	41	53	63	78	82	99	118	120							
19-12.12	7	11	19	29	41	53	63	78	82	95	99	120							
19-12.13	7	11	19	25	41	53	63	78	82	95	100	120							
19-12.14	7	11	21	41	55	58	78	86	92	99	101	120							
19-12.15	7	11	19	38	57	60	73	85	92	99	118	120							
19-12.16	7	11	21	38	57	76	83	90	111	118	120	123							
19-12.17a	7	11	21	25	38	41	58	78	84	101	107	120							
19-12.17b	7	11	19	29	38	57	60	73	85	106	118	120							
19-12.19	7	11	21	25	38	44	58	77	83	101	118	120							
19-12.20a	7	11	19	29	38	41	60	69	91	106	116	120							
19-12.20b	7	11	19	29	38	41	55	73	85	108	118	120							
19-12.161	7	11	19	29	35	41	55	73	87	102	108	120							
19-12.640	7	11	21	38	57	76	87	93	98	107	118	120							
19-12.3218	7	11	19	25	26	28	35	45	53	78	118	120							
19-12.6923	7	11	19	25	26	28	35	45	50	86	110	120							
19-12.12482	7	11	19	25	26	28	31	35	45	86	110	120							
19-12.12483	7	11	19	21	25	26	28	35	45	86	110	120							
19-12.14059	7	11	19	25	26	28	31	35	45	77	118	120							
19-12.18529	7	21	28	38	44	59	79	81	98	112	121	122							
19-12.26380a	7	11	14	19	25	26	28	31	45	77	110	120							
19-12.26380b	7	11	14	19	25	26	28	31	45	77	117	120							
19-12.27425	7	11	13	19	21	22	25	26	46	92	103	120							
19-12.31264	7	11	13	19	21	22	25	26	46	78	118	120							
19-12.31266	7	11	19	21	25	26	28	31	45	77	117	120							
19-12.38700	7	27	43	51	56	75	83	88	99	104	112	125							

k = 20, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)			alp			df		C2FI		Lmax		df		C2FI		Lmax		CD2*		CD2	
20-13.1	36	152	340	1	24	60	14	1	0	0	0	0	0	119	24	4	111	28084	1	15.6994	1	1
20-13.2	38	156	310	2	41	39	21	2	0	0	0	0	0	123	41	4	6	715	2	15.7043	2	2
20-13.3	39	152	308	3	40	39	20	3	0	0	0	0	0	122	40	4	11	1032	3	15.7056	3	3
20-13.4	39	152	308	3	38	39	22	2	0	0	0	0	0	121	38	4	26	1929	4	15.7056	3	3
20-13.5	40	148	316	5	34	42	20	3	0	0	0	0	0	119	34	4	111	5165	6	15.7072	5	5
20-13.6	40	148	316	5	30	54	8	7	0	0	0	0	0	119	30	4	111	11873	5	15.7072	5	5
20-13.7	40	152	308	7	36	42	18	4	0	0	0	0	0	120	36	4	54	3164	7	15.7080	7	7
20-13.8	40	153	300	8	39	39	19	4	0	0	0	0	0	121	39	4	27	1501	8	15.7080	8	8
20-13.9	40	154	298	9	40	42	14	6	0	0	0	0	0	122	40	4	12	1033	9	15.7082	9	9
20-13.10	40	154	298	9	39	39	19	4	0	0	0	0	0	121	39	4	28	1502	10	15.7082	10	10
20-13.11	40	156	296	11	31	60	4	3	3	0	0	0	0	121	31	5	29	9924	485	15.7087	12	12
20-13.12	40	156	300	12	40	42	14	6	0	0	0	0	0	122	40	4	13	1034	11	15.7088	13	13
20-13.13	41	144	312	13	32	45	16	5	0	0	0	0	0	118	32	4	230	7874	12	15.7085	11	11
20-13.14	41	150	301	14	41	36	19	5	0	0	0	0	0	121	41	4	30	716	13	15.7097	16	16
20-13.15	41	150	301	15	36	39	20	4	0	0	0	0	0	119	36	4	115	3165	15	15.7097	14	14
20-13.16	41	150	301	15	35	42	17	5	0	0	0	0	0	119	35	4	114	4168	14	15.7097	14	14
20-13.17	41	152	294	17	43	35	20	3	1	0	0	0	0	122	43	5	14	415	486	15.7100	17	17
20-13.18	41	152	294	17	39	36	21	4	0	0	0	0	0	120	39	4	55	1503	16	15.7100	17	17
20-13.19	41	152	295	19	39	36	21	4	0	0	0	0	0	120	39	4	55	1504	17	15.7100	19	19
20-13.20	41	152	296	20	46	27	26	3	0	0	0	0	0	122	46	4	15	190	18	15.7100	20	20
20-13.21	41	152	296	20	36	45	12	7	0	0	0	0	0	120	36	4	57	3166	19	15.7100	20	20

k = 20, Designs sorted based on degrees of freedom used

Design	wlp(w_4, \dots)			wlp rank	alp			df		C2FI	lmax	CD2*	CD2 rank									
										rank	rank		rank									
20-13.23128	64	128	280	23128	72	18	1	0	12	2	1	0	0	0	126	72	7	1	4	47887	15.7578	30523
20-13.47458	80	112	280	47458	84	6	1	0	0	14	1	0	0	0	126	84	7	2	1	51633	15.7915	55382
20-13.7545	54	148	266	7545	59	30	1	12	0	1	2	0	0	0	125	59	7	3	13	45588	15.7390	12963
20-13.58	42	154	284	58	46	39	13	3	3	0	0	0	0	0	124	46	5	4	191	497	15.7126	61
20-13.16206	60	132	272	16206	70	6	13	12	0	0	3	0	0	0	124	70	7	5	5	46802	15.7491	23100
20-13.2	38	156	310	2	41	39	21	2	0	0	0	0	0	0	123	41	4	6	715	2	15.7043	2
20-13.62	42	156	286	62	50	23	27	2	1	0	0	0	0	0	123	50	5	7	64	501	15.7131	74
20-13.63	42	156	286	62	38	55	3	2	5	0	0	0	0	0	123	38	5	8	1933	502	15.7131	73

k = 20, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	lmax	df	C2FI	lmax	CD2*	CD2 rank									
20-13.47458	80	112	280	47458	84	6	1	0	0	14	1	0	0	0	126	84	7	2	1	51633	15.7915	55382
20-13.52497	84	108	256	52497	78	0	7	0	0	14	1	0	0	0	120	78	7	110	2	52866	15.7993	56241
20-13.50328	82	108	270	50328	75	9	2	0	0	14	1	0	0	0	121	75	7	53	3	52274	15.7950	55770
20-13.23128	64	128	280	23128	72	18	1	0	12	2	1	0	0	0	126	72	7	1	4	47887	15.7578	30523
20-13.16206	60	132	272	16206	70	6	13	12	0	0	3	0	0	0	124	70	7	5	5	46802	15.7491	23100
20-13.57639	108	84	336	57639	70	6	1	0	0	0	15	0	0	0	112	70	7	3369	6	55270	15.8520	57809

k = 20, Designs sorted based on minimizing Lmax

Design	wlp(w ₄ ,...)		wlp rank	alp		df		C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank								
20-13.1	36	152	340	1	24	60	14	1	0	0	0	0	0	0	119	24	4	111	28084	1	15.6994	1
20-13.2	38	156	310	2	41	39	21	2	0	0	0	0	0	0	123	41	4	6	715	2	15.7043	2
20-13.3	39	152	308	3	40	39	20	3	0	0	0	0	0	0	122	40	4	11	1032	3	15.7056	3
20-13.4	39	152	308	3	38	39	22	2	0	0	0	0	0	0	121	38	4	26	1929	4	15.7056	3
20-13.6	40	148	316	5	30	54	8	7	0	0	0	0	0	0	119	30	4	111	11873	5	15.7072	5
20-13.5	40	148	316	5	34	42	20	3	0	0	0	0	0	0	119	34	4	111	5165	6	15.7072	5
20-13.7	40	152	308	7	36	42	18	4	0	0	0	0	0	0	120	36	4	54	3164	7	15.7080	7
20-13.8	40	153	300	8	39	39	19	4	0	0	0	0	0	0	121	39	4	27	1501	8	15.7080	8
20-13.9	40	154	298	9	40	42	14	6	0	0	0	0	0	0	122	40	4	12	1033	9	15.7082	9
20-13.10	40	154	298	9	39	39	19	4	0	0	0	0	0	0	121	39	4	28	1502	10	15.7082	10

k = 20, Design generators

Design	Design Generators														
20-13.1	7	11	21	41	54	58	79	86	92	99	101	120	123		
20-13.2	7	11	21	38	60	70	73	82	95	101	107	118	120		
20-13.3	7	11	19	38	57	60	73	84	93	99	110	118	120		
20-13.4	7	11	19	38	57	60	73	85	92	99	110	118	120		
20-13.5	7	11	19	29	38	57	60	73	85	95	106	118	120		
20-13.6	7	11	14	19	38	57	60	73	85	95	101	106	120		
20-13.7	7	11	21	25	38	41	55	58	78	84	101	107	120		
20-13.8	7	11	21	25	38	55	58	78	84	93	101	107	120		
20-13.9	7	11	13	21	38	57	76	83	90	101	111	118	120		
20-13.10	7	11	19	29	38	57	60	73	85	91	106	118	120		
20-13.11	7	11	19	30	35	41	73	84	93	101	111	114	120		
20-13.12	7	11	19	29	41	47	49	55	91	94	99	102	120		
20-13.13	7	11	19	38	57	60	73	85	95	101	106	119	120		
20-13.14	7	11	21	38	57	63	76	83	90	111	118	120	123		
20-13.15	7	11	19	29	38	57	60	73	85	99	110	118	120		
20-13.16	7	11	21	25	38	41	58	78	82	84	101	107	120		
20-13.17	7	11	13	19	38	57	60	73	85	92	99	118	120		
20-13.18	7	11	21	38	59	73	81	82	95	99	108	117	120		
20-13.19	7	11	19	29	38	57	60	70	73	99	110	118	120		
20-13.20	7	11	19	30	38	41	52	59	74	85	111	118	120		
20-13.21	7	11	19	30	41	49	52	61	74	87	101	111	120		
20-13.58	7	11	19	29	41	55	62	74	84	102	108	111	120		
20-13.62	7	11	19	29	30	41	53	63	78	82	95	99	120		
20-13.63	7	11	19	25	26	41	53	63	78	82	95	100	120		
20-13.7545	7	11	19	25	26	28	31	35	45	53	86	110	120		
20-13.16206	7	11	19	25	26	28	31	35	45	59	86	110	120		
20-13.23128	7	11	19	21	25	26	28	31	35	45	86	110	120		
20-13.47458	7	11	14	19	22	25	26	28	31	35	45	86	110	120	
20-13.50328	7	11	13	19	21	22	25	26	28	31	45	77	117	120	
20-13.52497	7	11	14	19	21	25	26	28	31	45	77	117	120		
20-13.57639	7	27	43	51	56	75	83	88	99	104	112	123	125		

k = 21, Designs sorted based on word length pattern

Design	wlp(w ₁ ,...)			wlp			alp			df C2FI			Lmax			df C2FI			Lmax			CD2*			CD2			
21-14.1	51	200	414	1	26	54	15	4	3	0	0	0	0	0	0	0	0	0	0	0	123	26	5	23	17819	45	14.1759	2
21-14.2	51	202	400	2	28	51	12	11	0	0	0	0	0	0	0	0	0	0	0	0	123	28	4	24	10484	1	14.1761	3
21-14.3	52	184	452	3	24	48	18	9	0	0	0	0	0	0	0	0	0	0	0	0	120	24	4	244	25188	2	14.1753	1
21-14.4	52	194	420	4	31	38	26	5	1	0	0	0	0	0	0	0	0	0	0	0	122	31	5	57	5419	46	14.1768	4
21-14.5	52	196	412	5	33	38	24	6	1	0	0	0	0	0	0	0	0	0	0	0	123	33	5	25	3019	47	14.1771	5
21-14.6	52	196	416	6	36	36	22	9	0	0	0	0	0	0	0	0	0	0	0	0	124	36	4	9	1156	3	14.1772	7
21-14.7	52	198	402	7	35	39	19	10	0	0	0	0	0	0	0	0	0	0	0	0	124	35	4	10	1882	4	14.1774	8
21-14.8	52	201	400	8	36	48	16	0	6	0	0	0	0	0	0	0	0	0	0	0	127	36	5	1	1157	48	14.1780	9
21-14.9	53	184	440	9	8	66	12	7	0	1	0	0	0	0	0	0	0	0	0	0	115	8	6	1698	40852	8560	14.1772	6
21-14.10	53	190	422	10	34	39	18	11	0	0	0	0	0	0	0	0	0	0	0	0	123	34	4	26	2365	5	14.1781	10
21-14.11	53	190	422	10	32	39	20	10	0	0	0	0	0	0	0	0	0	0	0	0	122	32	4	58	3933	6	14.1781	10
21-14.12	53	192	412	12	32	45	12	13	0	0	0	0	0	0	0	0	0	0	0	0	123	32	4	27	3934	7	14.1783	12
21-14.13	53	193	413	13	34	37	24	5	2	0	0	0	0	0	0	0	0	0	0	0	123	34	5	28	2366	49	14.1786	13
21-14.14	53	193	413	13	29	41	22	7	1	0	0	0	0	0	0	0	0	0	0	0	121	29	5	120	8257	50	14.1786	13
21-14.15	53	194	405	15	36	36	24	6	0	1	0	0	0	0	0	0	0	0	0	0	124	36	6	11	1158	8561	14.1787	15
21-14.16	53	195	401	16	37	35	22	8	1	0	0	0	0	0	0	0	0	0	0	0	124	37	5	12	914	51	14.1788	16
21-14.17	53	196	404	17	41	33	24	4	3	0	0	0	0	0	0	0	0	0	0	0	126	41	5	6	240	52	14.1791	18
21-14.18	53	196	404	17	34	39	18	11	0	0	0	0	0	0	0	0	0	0	0	0	123	34	4	29	2367	8	14.1791	17
21-14.19	53	199	395	19	29	47	14	10	1	0	0	0	0	0	0	0	0	0	0	0	122	29	5	59	8258	53	14.1795	19
21-14.20	53	200	400	20	20	72	0	7	0	3	0	0	0	0	0	0	0	0	0	0	123	20	6	30	34343	8562	14.1799	22
21-14.21	54	186	438	21	32	35	25	7	1	0	0	0	0	0	0	0	0	0	0	0	121	32	5	121	3935	54	14.1796	20

k = 21, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)		wlp rank	alp				df		C2FI	Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank	
21-14.8	52	201 400	8	36	48	16	0	6	0	0	0	0	1	1157	48	14.1780	9
21-14.110	56	189 392	110	56	0	49	0	0	0	1	0	0	2	8	51208	14.1836	113
21-14.2560	64	181 392	2560	62	0	37	0	6	0	1	0	0	3	3	51401	14.1986	4100
21-14.23744	80	165 392	23744	72	0	19	0	12	0	3	0	0	4	2	56822	14.2290	7684
21-14.80683	112	133 392	80683	84	0	7	0	0	0	15	0	0	5	1	74585	14.2896	82077
21-14.17	53	196 404	17	41	33	24	4	3	0	0	0	0	6	240	52	14.1791	18
21-14.225	57	196 376	225	44	30	24	4	0	3	0	0	0	7	79	8605	14.1869	331
21-14.7379	69	196 364	7379	54	23	12	13	0	0	2	1	0	8	10	75434	14.2121	16832
21-14.6	52	196 416	6	36	36	22	9	0	0	0	0	0	9	1156	3	14.1772	7
21-14.7	52	198 402	7	35	39	19	10	0	0	0	0	0	10	1882	4	14.1774	8

k = 21, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w_4, \dots)		wlp rank	alp			df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank					
										rank	rank	rank		rank					
21-14.80683	112	133	392	80683	84	0	7	0	0	15	0	0	7	5	1	74585	14.2896	82077	
21-14.23744	80	165	392	23744	72	0	19	0	12	0	3	0	0	4	2	56822	14.2290	37684	
21-14.2560	64	181	392	2560	62	0	37	0	6	0	1	0	0	3	3	51401	14.1986	4100	
21-14.18122	77	164	404	18122	62	9	6	19	0	6	0	0	0	56	4	17698	14.2227	28548	
21-14.41505	93	148	372	41505	62	17	6	1	0	14	0	1	0	8	119	5	77854	14.2523	74961
21-14.38737	92	148	380	38737	60	20	6	0	0	14	0	1	0	8	118	6	77767	14.2503	73985
21-14.29904	84	153	384	29904	57	8	17	0	13	0	3	0	0	7	617	7	58605	14.2346	51023
21-14.110	56	189	392	110	56	0	49	0	0	0	1	0	0	2	8	51208	14.1836	113	
21-14.28450	83	153	391	28450	55	12	14	2	12	0	3	0	0	9	616	9	58086	14.2326	46493
21-14.7379	69	196	364	7379	54	23	12	13	0	0	2	1	0	8	10	75434	14.2121	16832	

k = 21, Designs sorted based on minimizing Lmax

Design	wlp(w ₄ ,...)		wlp rank	alp		df		C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank					
21-14.2	51	202	400	2	28	51	12	11	0	0	0	0	0	0	24	10484	1	14.1761	3
21-14.3	52	184	452	3	24	48	18	9	0	0	0	0	0	0	244	25188	2	14.1753	1
21-14.6	52	196	416	6	36	36	22	9	0	0	0	0	0	0	9	1156	3	14.1772	7
21-14.7	52	198	402	7	35	39	19	10	0	0	0	0	0	0	10	1882	4	14.1774	8
21-14.10	53	190	422	10	34	39	18	11	0	0	0	0	0	0	26	2365	5	14.1781	10
21-14.11	53	190	422	10	32	39	20	10	0	0	0	0	0	0	58	3933	6	14.1781	10
21-14.12	53	192	412	12	32	45	12	13	0	0	0	0	0	0	27	3934	7	14.1783	12
21-14.18	53	196	404	17	34	39	18	11	0	0	0	0	0	0	29	2367	8	14.1791	17

k = 21, Design generators

Design	Design Generators																				
21-14.1	7	14	25	42	54	61	69	88	104	112	121	122	124	127							
21-14.2	7	30	35	38	41	52	81	82	104	112	121	122	124	127							
21-14.3	7	29	30	35	37	41	44	70	73	104	112	121	122	124							
21-14.4	7	11	19	29	35	42	69	73	81	92	108	119	120	126							
21-14.5	7	11	19	29	35	38	52	73	101	104	112	121	122	124							
21-14.6	7	11	30	35	49	76	84	88	104	107	112	121	122	124							
21-14.7	7	11	13	19	21	22	25	35	61	62	78	84	111	120							
21-14.8	7	11	13	19	35	69	70	81	82	87	98	108	118	120							
21-14.9	7	11	19	25	26	59	95	97	98	104	112	121	122	124							
21-14.10	7	11	21	35	46	52	61	79	81	104	112	121	122	124							
21-14.11	7	11	19	29	35	45	53	57	70	73	74	94	108	120							
21-14.12	7	19	25	28	31	38	55	62	84	97	112	121	122	124							
21-14.13	7	11	19	29	38	41	49	55	69	74	76	111	120	126							
21-14.14	7	11	19	29	35	45	53	57	63	73	74	81	119	120							
21-14.15	7	11	21	26	50	56	59	61	95	104	112	121	122	124							
21-14.16	7	11	19	25	38	41	52	62	67	73	82	92	109	120							
21-14.17	7	11	19	35	38	41	42	55	59	73	74	93	101	120							
21-14.18	7	22	35	38	41	50	55	56	101	104	112	121	122	124							
21-14.19	7	35	41	42	52	67	87	102	104	112	121	122	124	127							
21-14.20	7	11	19	28	31	35	49	76	85	104	112	121	122	124							
21-14.21	7	11	35	38	42	49	50	76	101	104	112	121	122	124							
21-14.110	7	11	21	35	46	52	69	73	76	104	112	121	122	124							
21-14.225	7	19	25	28	31	38	44	50	55	81	112	121	122	124							
21-14.2560	7	11	19	29	38	41	55	67	74	76	84	109	118	120							
21-14.7379	7	11	19	21	28	31	38	41	52	104	112	121	122	124							
21-14.18122	7	11	19	29	38	41	60	69	90	95	111	119	120	123							
21-14.23744	7	35	38	41	42	49	52	63	82	104	112	121	122	124							
21-14.28450	7	19	25	26	28	38	52	79	81	109	112	121	122	124							
21-14.29904	7	11	21	31	38	77	94	103	104	112	121	122	124	127							
21-14.38737	7	11	25	26	31	41	53	91	104	112	115	121	122	124							
21-14.41505	7	11	13	19	21	31	47	50	76	100	112	121	122	124							
21-14.80683	7	19	25	28	41	50	63	73	82	93	112	121	122	124							

k = 22, Designs sorted based on word length pattern

Design	wlp(w _i ,...)		wlp rank		alp		df		C2FI		Lmax		df		C2FI		Lmax		rank		CD2*		CD2 rank	
22-15.1	65	248	572	1	25	36	32	8	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.2	65	256	552	2	12	68	12	6	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.3	66	254	544	3	21	52	12	15	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.4	67	248	564	4	24	43	23	9	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.5	68	240	568	5	8	58	18	7	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.6	68	240	570	6	24	36	27	12	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.7	68	241	568	7	28	34	30	5	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.8	68	248	542	8	29	38	22	10	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.9	68	248	553	9	32	30	28	10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.10	68	248	553	9	20	46	22	7	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.11	68	249	544	11	4	70	6	11	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.12	68	249	548	12	29	32	30	7	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.13	68	253	536	13	21	50	14	12	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.14	68	256	521	14	9	80	0	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.15	68	256	530	15	17	54	16	7	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.16	69	236	578	16	28	40	15	17	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.17	69	240	552	17	17	45	24	10	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.18	69	240	562	18	25	36	25	11	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.19	69	240	562	19	30	32	26	12	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.20	69	242	548	20	17	46	22	10	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22-15.21	69	242	558	21	32	33	22	13	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

k = 22, Design generators

Design	Design Generators														
22-15.1	7	11	19	29	37	41	55	59	74	82	84	102	108	120	126
22-15.2	7	11	19	30	38	41	52	61	74	87	93	101	111	114	120
22-15.3	7	11	19	30	38	41	59	61	74	85	92	98	111	118	120
22-15.4	7	11	19	29	37	41	47	49	55	69	91	94	99	120	125
22-15.5	7	11	19	41	52	62	73	82	84	94	99	101	111	113	120
22-15.6	7	11	19	38	41	50	60	63	69	91	93	106	117	118	120
22-15.7	7	11	19	29	38	41	60	70	76	82	99	109	117	118	120
22-15.8	7	11	19	22	38	41	60	67	78	82	95	109	113	119	120
22-15.9	7	11	21	28	38	57	76	83	90	95	101	111	118	120	123
22-15.10	7	11	19	29	37	41	47	59	77	78	84	91	102	119	120
22-15.11	7	11	19	29	37	41	50	60	63	69	73	82	99	102	120
22-15.12	7	11	19	29	30	38	41	49	60	78	82	95	109	119	120
22-15.13	7	11	21	28	38	57	63	76	83	90	95	111	118	120	123
22-15.14	7	11	19	29	35	45	52	55	67	73	74	86	108	114	120
22-15.15	7	11	21	28	38	57	63	69	76	83	90	95	111	118	120
22-15.16	7	11	19	38	57	60	70	73	76	84	93	99	110	118	120
22-15.17	7	11	19	29	37	41	50	60	69	73	82	95	102	120	126
22-15.18	7	11	19	38	41	55	59	73	76	85	86	91	103	113	120
22-15.19	7	11	19	29	37	41	49	59	77	78	84	87	99	106	120
22-15.20	7	11	19	29	35	45	53	73	79	81	87	103	118	120	123
22-15.21	7	11	19	29	38	41	50	55	73	85	92	106	108	118	120
22-15.22	7	11	13	19	22	38	57	60	73	85	92	99	106	118	120
22-15.23	7	11	19	29	38	41	50	55	73	85	92	99	108	118	120
22-15.26	7	11	19	29	38	41	55	62	67	73	87	108	114	120	123
22-15.39	7	11	19	29	35	45	53	59	70	73	81	87	103	120	126
22-15.43	7	11	19	29	37	41	49	55	59	70	87	89	90	116	120
22-15.46	7	11	13	21	28	38	42	57	76	83	90	97	111	118	120
22-15.4645	7	11	19	29	30	35	41	42	44	47	53	59	78	118	120
22-15.8501	7	11	19	29	38	41	47	70	73	79	99	109	110	117	120
22-15.29288	7	11	19	21	22	25	26	28	31	35	45	46	77	118	120
22-15.30203	7	11	19	21	22	25	26	28	31	35	45	67	77	118	120
22-15.30206	7	11	19	21	22	25	26	28	31	35	45	46	77	117	120

k = 23, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)			wlp			alp			df		C2FI	df		C2FI	Lmax		CD2*	CD2				
												rank		rank	rank		rank	rank	rank				
23-16.1	83	316	744	1	12	52	24	9	2	2	1	0	0	0	0	125	12	7	10	32307	5495	11.5703	1
23-16.2	83	318	734	2	14	54	11	17	6	0	0	0	0	0	0	125	14	5	11	31330	1	11.5704	2
23-16.3	84	312	744	3	0	58	26	1	11	0	0	0	0	0	0	119	0	5	472	34743	2	11.5713	3
23-16.4	84	319	726	4	12	54	16	10	9	0	0	0	0	0	0	124	12	5	51	32308	3	11.5722	6
23-16.5	85	304	744	5	9	49	20	16	2	2	0	0	0	0	0	121	9	6	249	33463	61	11.5716	4
23-16.6	85	306	756	6	25	26	34	12	4	1	0	0	0	0	0	125	25	6	12	9682	62	11.5721	5
23-16.7	85	312	730	7	22	38	26	9	7	1	0	0	0	0	0	126	22	6	1	22091	63	11.5727	8
23-16.8	85	318	718	8	17	44	26	8	4	3	0	0	0	0	0	125	17	6	13	29653	64	11.5736	15
23-16.9	86	299	766	9	20	32	29	14	4	1	0	0	0	0	0	123	20	6	95	26421	65	11.5727	7
23-16.10	86	304	753	10	18	37	27	10	8	0	0	0	0	0	0	123	18	5	96	28456	4	11.5734	13
23-16.11	86	305	740	11	4	53	23	6	10	0	0	0	0	0	0	119	4	5	473	34497	5	11.5734	10
23-16.12	86	305	740	12	6	46	31	4	8	1	0	0	0	0	0	119	6	6	474	34227	66	11.5734	10
23-16.13	86	305	740	13	0	64	13	10	8	1	0	0	0	0	0	119	0	6	475	34744	67	11.5734	10
23-16.14	86	306	735	14	10	48	21	12	6	1	0	0	0	0	0	121	10	6	250	32981	68	11.5735	14
23-16.15	86	308	728	15	23	35	30	8	4	3	0	0	0	0	0	126	23	6	2	18285	69	11.5737	16
23-16.16	86	320	697	16	7	66	16	6	2	4	0	1	0	0	0	125	7	8	14	34110	20195	11.5753	26
23-16.17	86	324	696	17	13	45	36	0	0	7	0	0	0	0	0	124	13	6	52	32028	70	11.5760	31
23-16.18	87	290	790	18	22	42	11	21	6	0	0	0	0	0	0	125	22	5	15	22092	6	11.5732	9

k = 23, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ , ...)			alp			df C2FI			C2FI			Lmax			CD2*	CD2 rank									
	wlp	rank		wlp	rank		wlp	rank		df	C2FI	rank	df	C2FI	rank											
23-16.7	85	312	730	7			22	38	26	9	7	1	0	0	0	0	0	0	126	22	6	1	22091	63	11.5727	8
23-16.15	86	308	728	15			23	35	30	8	4	3	0	0	0	0	0	0	126	23	6	2	18285	69	11.5737	16
23-16.21	87	300	754	21			26	31	29	8	8	1	0	0	0	0	0	0	126	26	6	3	6412	73	11.5744	21
23-16.29	88	300	745	29			28	30	21	21	0	3	0	0	0	0	0	0	126	28	6	4	2668	76	11.5759	30
23-16.31	88	305	724	31			24	38	21	12	7	0	1	0	0	0	0	0	126	24	7	5	13130	5497	11.5765	39
23-16.47	89	298	728	47			26	35	22	11	7	2	0	0	0	0	0	0	126	26	6	6	6415	91	11.5770	49
23-16.123	92	292	725	123			30	33	11	25	0	4	0	0	0	0	0	0	126	30	6	7	1289	145	11.5810	155
23-16.124	92	300	717	124			27	31	29	10	1	4	0	1	0	0	0	0	126	27	8	8	4520	20200	11.5823	231
23-16.537	97	270	776	537			36	27	8	25	3	4	0	0	0	0	0	0	126	36	6	9	208	338	11.5863	646
23-16.1	83	316	744	1			12	52	24	9	2	2	1	0	0	0	0	0	125	12	7	10	32307	5495	11.5703	1

k = 23, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp			df C2FI			df	C2FI	Lmax	rank	CD2*	CD2 rank						
23-16.9896	115 244 740	9896	45	6	27	4	15	0	0	3	0	0	123	45	8	189	1	23298	11.6118	18586
23-16.32406	140 140 1109	32406	44	18	0	1	12	17	1	0	0	0	116	44	7	3056	2	19930	11.6418	32819
23-16.32595	141 138 1102	32595	44	18	0	2	12	14	3	0	0	0	116	44	7	3057	3	19948	11.6430	32929
23-16.32597	141 138 1104	32597	44	18	0	0	17	11	2	1	0	0	116	44	8	3058	4	29387	11.6431	32933
23-16.32747	142 138 1095	32747	44	18	0	5	6	17	3	0	0	0	116	44	7	3059	5	19968	11.6446	33033
23-16.32751	142 138 1095	32751	44	18	0	3	12	11	5	0	0	0	116	44	7	3060	6	19972	11.6446	33036

k = 23, Design generators

Design	Design Generators																						
23-16.1	7	11	19	25	26	31	35	45	46	77	81	92	100	106	118	120							
23-16.2	7	11	19	30	38	57	60	70	73	76	84	93	99	110	118	120							
23-16.3	7	11	19	29	37	59	62	73	87	94	99	106	111	117	118	120							
23-16.4	7	11	19	29	41	47	49	59	62	77	82	92	97	110	116	120							
23-16.5	7	11	19	29	35	46	53	57	73	76	82	100	109	118	120	123							
23-16.6	7	11	19	29	37	41	47	55	59	74	82	84	102	108	120	126							
23-16.7	7	11	19	29	37	41	49	55	59	70	76	87	89	90	116	120							
23-16.8	7	11	19	29	37	38	41	50	60	63	69	73	91	106	113	120							
23-16.9	7	11	21	26	28	38	57	63	73	82	95	99	110	119	120	125							
23-16.10	7	11	19	29	37	41	47	59	77	78	84	91	99	102	119	120							
23-16.11	7	11	19	29	37	41	50	60	63	69	73	82	99	102	106	120							
23-16.12	7	11	19	29	37	41	55	59	77	78	82	87	91	99	116	120							
23-16.13	7	11	19	22	35	38	57	60	63	73	87	93	103	109	114	120							
23-16.14	7	11	19	29	37	41	47	55	59	82	99	109	110	113	116	120							
23-16.15	7	11	19	29	30	38	41	47	70	84	89	90	99	106	108	120							
23-16.16	7	11	19	25	26	35	45	53	67	78	86	92	100	103	106	120							
23-16.17	7	11	19	29	35	37	41	50	60	63	73	87	94	102	111	120							
23-16.18	7	11	19	38	57	60	70	73	76	81	84	93	99	110	118	120							
23-16.21	7	11	19	29	37	41	49	55	59	70	87	89	90	106	116	120							
23-16.22	7	11	19	29	37	41	47	49	55	62	77	82	92	97	116	120							
23-16.24	7	11	19	30	38	41	44	49	59	69	76	93	97	111	117	120							
23-16.29	7	11	21	28	38	42	57	76	83	90	95	101	111	118	120	123							
23-16.31	7	11	19	29	37	41	59	73	76	79	85	91	99	109	113	120							
23-16.47	7	11	19	29	37	41	50	59	73	76	79	85	99	109	113	120							
23-16.123	7	11	19	29	35	37	41	55	73	74	76	82	94	102	116	120							
23-16.124	7	11	19	25	26	28	35	45	53	54	67	73	86	103	114	120							
23-16.537	7	11	19	29	35	37	41	55	59	73	74	76	82	94	102	120							
23-16.9896	7	11	13	14	19	25	26	31	35	41	53	67	73	85	100	120							
23-16.32406	7	11	13	14	19	21	38	41	44	50	55	61	62	93	101	120							
23-16.32595	7	11	13	19	21	25	26	35	38	41	44	50	55	93	101	120							
23-16.32597	7	11	13	19	25	26	28	35	41	44	47	61	62	78	118	120							
23-16.32747	7	11	13	14	19	25	28	54	86	104	110	117	121	122	124	127							
23-16.32751	7	11	13	19	21	25	35	38	41	44	47	50	55	93	101	120							

k = 24, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)	wlp rank	alp			df	C2FI	Lmax	df	C2FI	Lmax	rank	CD2*	CD2 rank
24-17.1	102 384 992	1	0	54	16 24	0	4	0	0	0	0	0	10.4617	1
24-17.2	102 394 985	2	7	57	9 17 12	0	0	0	0	0	0	0	10.4631	2
24-17.3	103 393 972	3	14	39 31	7 9 3	0	0	0	0	0	0	0	10.4643	3
24-17.4	104 392 960	4	15	36 33 12	0 7 0	0	0	0	0	0	0	0	10.4655	7
24-17.5	105 372 1026	5	15	29 32 15	7 2 0	0	0	0	0	0	0	0	10.4648	4
24-17.6	105 374 1008	6	3	37 40	3 11	2	0	0	0	0	0	0	10.4649	5
24-17.7	105 378 988	7	5	45 24 13	9 2 0	0	0	0	0	0	0	0	10.4653	6
24-17.8	105 400 930	8	4	53 32	6 0 4	2	1	0	0	0	0	0	10.4679	11
24-17.9	105 405 928	9	8	42 42	3 0 3	4	0	0	0	0	0	0	10.4687	15
24-17.10	106 374 1000	10	0	47 29	9 7 4	0	0	0	0	0	0	0	10.4663	8
24-17.11	107 370 994	11	9	38 27 12 10	2 0 0	0	0	0	0	0	0	0	10.4671	10
24-17.12	107 380 988	12	12	48 12 24	3 0 3	0	0	0	0	0	0	0	10.4686	14
24-17.13	108 352 1072	13	16	48 0 26 12	0 0 0	0	0	0	0	0	0	0	10.4668	9
24-17.14	108 367 996	14	0	53 18 12 10	3 0 0	0	0	0	0	0	0	0	10.4682	12
24-17.15	108 370 987	15	10	37 28	8 14 1	0	0	0	0	0	0	0	10.4686	13
24-17.16	108 373 1012	16	16	48 0 26 12	0 0 0	0	0	0	0	0	0	0	10.4693	18
24-17.17	109 363 1000	17	13	31 29 14	8 3 0	0	0	0	0	0	0	0	10.4692	16
24-17.18	109 366 1006	18	17	30 28 13	9 3 0	0	0	0	0	0	0	0	10.4697	22
24-17.19	109 367 988	19	4	43 26 11	8 4 0	0	0	0	0	0	0	0	10.4696	19
24-17.20	109 367 988	19	3	44 27 11	7 3 1	0	0	0	0	0	0	0	10.4696	19

k = 24, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp										df C2FI		Lmax	C2FI	CD2*	CD2 rank	
24-17.3	103 393 972	3	14	39	31	7	9	3	0	0	0	0	0	0	1	24313	5	10.4643	3
24-17.4	104 392 960	4	15	36	33	12	0	7	0	0	0	0	0	0	2	24068	6	10.4655	7
24-17.22	109 373 968	22	20	34	25	12	8	3	1	0	0	0	0	0	3	19896	1122	10.4703	25
24-17.35	111 364 996	35	24	30	16	27	3	0	3	0	0	0	0	0	4	9940	1126	10.4723	45
24-17.91	115 356 972	91	24	39	0	33	3	1	3	0	0	0	0	0	5	9941	1157	10.4768	133
24-17.94	115 364 964	94	24	26	32	11	4	4	1	0	0	0	0	0	6	9943	20625	10.4779	178
24-17.2	102 394 985	2	7	57	9	17	12	0	0	0	0	0	0	0	7	26967	1	10.4631	2
24-17.8	105 400 930	8	4	53	32	6	0	4	2	1	0	0	0	0	8	27392	10050	10.4679	11
24-17.9	105 405 928	9	8	42	42	3	0	3	4	0	0	0	0	0	9	26390	1118	10.4687	15
24-17.12	107 380 988	12	12	48	12	24	3	0	3	0	0	0	0	0	10	25053	1119	10.4686	14

k = 24, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ , ...)			alp			df C2FI			Lmax			CD2*		CD2								
			wlp rank										rank		rank								
24-17.28100	250	54	2304	28100	45	0	0	0	0	1	15	15	0	0	100	45	8	28068	1	20624	10.6570	28100	
24-17.28101a	251	53	2296	28101	45	0	0	0	0	1	17	12	0	1	100	45	10	28069	2	27802	10.6583	28101	
24-17.28101b	251	53	2296	28101	45	0	0	0	0	0	2	15	12	2	0	100	45	9	28069	2	26133	10.6583	28101
24-17.28101c	251	53	2296	28101	45	0	0	0	0	0	2	15	12	2	0	100	45	9	28069	2	26133	10.6583	28101
24-17.28104	251	54	2296	28104	45	0	0	0	0	0	2	15	12	2	0	100	45	9	28072	5	26135	10.6584	28104
24-17.28105	251	55	2296	28105	45	0	0	0	0	0	2	15	12	2	0	100	45	9	28073	6	26136	10.6586	28105
24-17.28106	251	56	2296	28106	45	0	0	0	0	0	1	17	12	0	1	100	45	10	28074	7	27803	10.6588	28106
24-17.28107	252	52	2288	28107	45	0	0	0	0	0	4	12	12	3	0	100	45	9	28075	8	26137	10.6595	28107

k = 24, Design generators

Design	Design Generators																							
24-17.1	7	11	19	29	35	46	53	57	73	76	82	87	100	109	118	120	123							
24-17.2	7	11	19	30	38	57	60	70	73	76	81	84	93	99	110	118	120							
24-17.3	7	11	19	29	41	47	49	59	62	77	82	92	97	110	116	119	120							
24-17.4	7	11	19	29	37	38	41	50	60	63	69	73	82	91	106	113	120							
24-17.5	7	11	21	26	28	38	57	63	73	76	82	95	99	110	119	120	125							
24-17.6	7	11	19	29	35	53	57	73	76	82	94	98	100	109	118	120	123							
24-17.7	7	11	19	29	35	46	53	57	73	76	82	94	100	109	118	120	123							
24-17.8	7	11	19	25	26	35	45	53	63	67	78	86	92	100	103	106	120							
24-17.9	7	11	19	29	35	37	41	50	60	63	73	87	94	102	111	113	120							
24-17.10	7	11	19	29	37	41	59	62	73	82	87	99	106	111	117	118	120							
24-17.11	7	11	19	29	38	41	44	55	62	69	76	89	90	98	111	120	125							
24-17.12	7	11	19	30	38	41	44	49	52	61	74	87	93	101	111	114	120							
24-17.13	7	11	19	38	57	60	70	73	76	81	84	91	93	99	110	118	120							
24-17.14	7	11	19	21	38	41	52	62	69	79	87	89	100	106	114	120	125							
24-17.15	7	11	19	29	37	41	47	55	59	62	82	99	109	110	113	116	120							
24-17.16	7	11	19	29	37	41	47	49	55	59	62	77	78	82	84	91	120							
24-17.17	7	11	19	29	37	41	49	55	59	62	77	78	91	97	98	111	120							
24-17.18	7	11	19	29	37	41	47	49	59	62	69	84	89	90	99	102	120							
24-17.19	7	11	19	29	37	41	44	62	73	76	87	99	106	111	117	118	120							
24-17.20	7	11	19	29	35	44	53	57	73	76	82	94	100	109	118	120	123							
24-17.22	7	11	19	29	37	38	41	50	60	63	69	73	76	82	91	113	120							
24-17.35	7	11	21	28	38	42	57	76	83	90	95	101	105	111	118	120	123							
24-17.91	7	11	19	29	35	37	41	55	59	73	74	76	82	94	102	116	120							
24-17.94	7	11	19	25	26	35	41	53	54	59	69	70	82	106	116	119	120							
24-17.28100	7	19	21	22	35	37	38	49	67	69	81	87	92	100	103	112	117							
24-17.28101a	7	19	21	22	35	37	38	49	55	67	81	84	95	100	103	112	117							
24-17.28101b	7	19	21	22	35	37	38	49	50	67	70	81	84	95	97	100	112							
24-17.28101c	7	19	21	22	35	37	38	49	67	69	81	82	87	97	111	112	118							
24-17.28104	7	19	21	22	35	37	38	49	55	67	69	81	84	95	100	103	112							
24-17.28105	7	19	21	22	35	37	38	49	67	81	87	92	100	103	112	115	117							
24-17.28106	7	19	21	22	35	37	38	49	67	81	82	87	92	100	103	112	115							
24-17.28107	7	19	21	22	35	37	38	49	63	67	81	82	84	87	97	112	117							

k = 25, Designs sorted based on word length pattern

Design	wlp(w ₁ ,...)		wlp rank	alp				df		C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank					
25-18.1	124	482	1312	1	0	64	0	18	20	0	0	0	0	127	0	5	1	20240	1	9.4697	1
25-18.2	125	504	1222	2	0	41	48	6	0	0	6	1	0	127	0	8	2	20241	3424	9.4730	3
25-18.3	126	468	1304	3	0	42	28	12	12	4	0	0	0	123	0	6	45	20242	2	9.4704	2
25-18.4	129	458	1310	4	5	33	34	7	15	4	0	0	0	123	5	6	46	19619	3	9.4732	4
25-18.5	130	449	1341	5	0	36	33	14	6	6	1	0	0	121	0	7	111	20243	98	9.4736	5
25-18.6	131	448	1324	6	9	26	35	12	10	6	0	0	0	123	9	6	47	18698	4	9.4747	6
25-18.7	132	449	1325	7	0	38	30	14	10	0	4	0	0	121	0	7	112	20244	99	9.4761	7
25-18.8	133	440	1350	8	15	24	29	17	9	5	1	0	0	125	15	7	19	16697	100	9.4765	8
25-18.9	133	442	1326	9	0	43	20	20	5	7	1	0	0	121	0	7	113	20245	101	9.4765	10
25-18.10	133	442	1326	10	3	34	29	17	5	7	1	0	0	121	3	7	115	19942	103	9.4765	9
25-18.11	133	442	1326	10	0	39	31	10	9	5	2	0	0	121	0	7	114	20246	102	9.4765	10
25-18.12	134	444	1280	12	0	54	16	0	24	4	0	0	0	123	0	6	48	20247	5	9.4777	12
25-18.13	135	432	1348	13	12	18	43	3	15	6	0	0	0	122	12	6	95	17805	6	9.4781	13
25-18.14	135	435	1320	14	3	36	29	12	6	10	0	0	0	121	3	6	117	19943	8	9.4782	14
25-18.15a	135	435	1320	14	0	30	35	15	3	10	0	0	0	118	0	6	706	20248	9	9.4782	14
25-18.15b	135	435	1320	14	0	45	20	15	6	10	0	0	0	121	0	6	116	20248	7	9.4782	14
25-18.17	135	442	1310	17	0	44	18	21	9	0	3	1	0	121	0	8	118	20250	3425	9.4791	17
25-18.18	135	442	1310	18	0	38	36	3	15	0	3	1	0	121	0	8	119	20251	3426	9.4791	18
25-18.19	136	432	1338	19	15	24	37	3	12	9	0	0	0	125	15	6	20	16698	10	9.4794	20
25-18.20	136	435	1317	20	3	39	20	21	6	4	3	0	0	121	3	7	120	19944	104	9.4796	21

k = 25, Designs sorted based on minimizing Lmax

Design	wlp(w ₄ ,...)			wlp rank			alp			df			C2FI			Lmax			CD2*			CD2 rank		
25-18.1	124	482	1312	1	0	64	0	18	20	0	0	0	0	0	127	0	5	1	20240	1	9.4697	1		
25-18.3	126	468	1304	3	0	42	28	12	12	4	0	0	0	0	123	0	6	45	20242	2	9.4704	2		
25-18.4	129	458	1310	4	5	33	34	7	15	4	0	0	0	0	123	5	6	46	19619	3	9.4732	4		
25-18.6	131	448	1324	6	9	26	35	12	10	6	0	0	0	0	123	9	6	47	18698	4	9.4747	6		
25-18.12	134	444	1280	12	0	54	16	0	24	4	0	0	0	0	123	0	6	48	20247	5	9.4777	12		
25-18.13	135	432	1348	13	12	18	43	3	15	6	0	0	0	0	122	12	6	95	17805	6	9.4781	13		
25-18.15b	135	435	1320	14	0	45	20	15	6	10	0	0	0	0	121	0	6	116	20248	7	9.4782	14		
25-18.14	135	435	1320	14	3	36	29	12	6	10	0	0	0	0	121	3	6	117	19943	7	9.4782	14		
25-18.15a	135	435	1320	14	0	30	35	15	3	10	0	0	0	0	118	0	6	706	20248	7	9.4782	14		
25-18.19	136	432	1338	19	15	24	37	3	12	9	0	0	0	0	125	15	6	20	16698	10	9.4794	20		

k = 25, Design generators

Design	Design Generators																								
25-18.1	7	11	19	29	37	41	47	49	55	59	62	77	78	82	84	91	102	120							
25-18.2	7	11	19	25	26	35	45	53	63	67	78	86	92	100	103	106	114	120							
25-18.3	7	11	19	29	35	46	53	57	60	73	76	82	87	100	109	118	120	123							
25-18.4	7	11	19	30	38	47	57	69	73	79	82	84	93	97	98	108	119	120							
25-18.5	7	11	13	19	21	38	41	55	59	70	73	87	91	99	101	106	116	120							
25-18.6	7	11	19	29	37	41	49	55	59	77	78	91	97	98	111	116	120	125							
25-18.7	7	11	19	29	37	41	55	59	77	78	82	87	91	99	102	106	116	120							
25-18.8	7	11	19	29	37	44	50	52	59	62	73	82	87	106	111	117	118	120							
25-18.9	7	11	19	29	37	41	59	73	76	82	87	94	99	106	111	117	118	120							
25-18.10	7	11	19	29	37	44	50	59	62	73	82	87	99	106	111	117	118	120							
25-18.11	7	11	13	19	31	38	41	55	59	70	73	87	91	99	101	106	116	120							
25-18.12	7	11	19	29	35	46	53	57	69	73	76	82	87	100	109	118	120	123							
25-18.13	7	11	19	30	38	47	52	57	69	73	79	82	84	93	97	98	119	120							
25-18.14	7	11	19	29	35	53	57	69	73	76	82	94	98	100	109	118	120	123							
25-18.15a	7	11	19	29	35	53	57	70	73	76	82	94	97	100	109	118	120	123							
25-18.15b	7	11	13	19	21	38	41	52	62	69	79	87	89	100	106	114	120	125							
25-18.17	7	11	19	29	37	50	59	62	73	76	82	87	91	99	106	111	117	118	120						
25-18.18	7	11	19	29	37	59	62	73	76	82	87	91	99	106	111	117	118	120							
25-18.19	7	11	19	29	30	38	41	50	60	78	82	87	91	100	106	117	118	120							
25-18.20	7	11	19	29	35	38	41	44	50	55	69	73	82	92	95	100	120	125							
25-18.27	7	11	19	29	30	38	57	60	70	89	92	99	109	110	117	118	120	123							
25-18.51	7	11	13	14	19	22	26	41	53	60	73	74	76	85	97	103	120	126							
25-18.59	7	11	21	26	28	42	44	51	77	78	95	104	107	112	118	121	122	124							
25-18.63	7	11	19	29	35	46	53	69	70	73	79	81	87	94	109	118	120	123							
25-18.134	7	11	13	14	19	35	38	44	57	58	69	81	82	87	93	106	111	120							
25-18.136	7	11	13	30	35	53	54	67	85	86	102	104	112	115	121	122	124	127							
25-18.137	7	11	19	22	25	26	28	31	35	45	46	67	77	78	117	118	120	123							
25-18.193	7	11	13	30	35	53	54	78	85	86	102	104	112	115	121	122	124	127							
25-18.874	7	11	19	29	35	45	52	58	67	69	70	73	74	79	81	97	118	120							
25-18.988	7	27	30	35	41	42	44	67	74	82	87	101	104	112	121	122	124	127							
25-18.1021	7	11	19	29	35	45	58	67	69	70	73	74	79	81	97	118	120	123							
25-18.1022	7	11	13	14	19	22	26	31	41	53	60	73	85	92	97	100	109	120							
25-18.2757	7	11	13	14	19	22	25	26	35	41	60	85	92	95	103	114	120	123							
25-18.20549a	7	19	21	22	35	37	38	49	55	67	69	81	84	95	100	103	112	117							
25-18.20549b	7	19	21	22	35	37	38	49	52	67	69	70	81	87	97	111	112	115							
25-18.20551	7	19	21	22	35	37	38	49	67	69	70	81	87	97	100	111	112	115							
25-18.20552	7	19	21	22	35	37	38	49	52	67	81	82	87	92	100	103	112	115							
25-18.20553	7	19	21	22	35	37	38	49	67	69	81	82	87	97	111	112	115	118							
25-18.20554	7	19	21	22	35	37	38	49	50	67	69	70	81	87	97	111	112	115							
25-18.20555	7	19	21	22	35	37	38	49	67	69	70	81	87	97	98	111	112	115							
25-18.20556	7	19	21	22	35	37	38	67	69	81	82	87	92	100	103	112	117	118							
25-18.20557	7	19	21	22	35	37	38	49	67	69	70	81	82	87	97	111	112	115							

k = 26, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)	wlp rank	alp										df	C2FI	Lmax	rank	CD2*	CD2 rank			
26-19.1	152 568 1704	1	0	29	41	4	16	8	0	0	0	0	0	0	124	0	6	13 13068	1	8.5797	1
26-19.2	155 555 1720	2	5	20	45	5	13	10	0	0	0	0	0	0	124	5	6	14 12525	2	8.5819	2
26-19.3	160 530 1767	3	0	30	30	20	6	5	5	0	0	0	0	0	122	0	7	43 13069	8	8.5854	3
26-19.4	161 530 1758	4	0	33	23	25	5	6	3	1	0	0	0	0	122	0	8	44 13070	708	8.5865	4
26-19.5	163 520 1783	5	15	18	27	19	15	0	6	0	0	0	0	0	126	15	7	3 10630	9	8.5879	5
26-19.6	163 523 1752	6	0	36	19	25	6	4	6	0	0	0	0	0	122	0	7	45 13071	10	8.5880	6
26-19.7	163 523 1752	7	3	30	19	31	3	4	6	0	0	0	0	0	122	3	7	46 12806	11	8.5880	6
26-19.8	164 523 1743	8	0	33	29	14	10	6	2	2	0	0	0	0	122	0	8	47 13072	709	8.5892	8
26-19.9	164 536 1664	9	0	42	28	0	12	15	1	0	0	0	0	0	124	0	7	15 13073	12	8.5900	9
26-19.10	166 516 1737	10	0	39	17	21	9	3	7	0	0	0	0	0	122	0	7	48 13074	13	8.5907	10
26-19.11	167 516 1728	11	0	42	8	30	6	6	1	3	0	0	0	0	122	0	8	49 13075	710	8.5918	12
26-19.12	168 492 1912	12	24	3	27	31	6	6	0	3	0	0	0	0	126	24	8	4 8959	711	8.5918	11
26-19.13	168 524 1672	13	5	33	32	2	9	14	3	0	0	0	0	0	124	5	7	16 12526	14	8.5935	15
26-19.14	169 490 1830	14	8	22	24	29	1	5	6	1	0	0	0	0	122	8	8	50 11866	712	8.5920	13
26-19.15	169 509 1722	15	3	33	24	14	12	2	8	0	0	0	0	0	122	3	7	51 12807	15	8.5933	14
26-19.16	170 506 1746	16	15	21	30	13	6	9	6	0	0	0	0	0	126	15	7	5 10631	16	8.5944	19
26-19.17	170 509 1725	17	0	42	16	15	13	4	4	2	0	0	0	0	122	0	8	52 13076	713	8.5946	21

k = 26, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)			alp			C2FI			C2FI			CD2*										
	wlp	rank		wlp	rank		df	C2FI	Lmax	rank	df	C2FI	Lmax	rank	CD2	rank							
26-19.224	181 468 1808	224	20	31	0	31	8	0	10	0	1	0	0	0	127	20	9	1	9213	4570	8.6037	370	
26-19.997	190 528 1520	997	0	72	0	0	0	28	0	0	0	0	0	0	1	127	0	13	2	13371	13476	8.6194	4435
26-19.5	163 520 1783	5	15	18	27	19	15	0	6	0	0	0	0	0	0	126	15	7	3	10630	9	8.5879	5
26-19.12	168 492 1912	12	24	3	27	31	6	6	0	3	0	0	0	0	0	126	24	8	4	8959	711	8.5918	11
26-19.16	170 506 1746	16	15	21	30	13	6	9	6	0	0	0	0	0	0	126	15	7	5	10631	16	8.5944	19
26-19.48	176 484 1848	48	24	3	30	34	0	0	3	6	0	0	0	0	0	126	24	8	6	8960	732	8.5999	113
26-19.49	176 486 1758	49	18	22	20	19	8	7	3	3	0	0	0	0	0	126	18	8	7	9448	733	8.5994	87
26-19.935	190 418 1978	935	24	21	18	10	9	9	6	3	0	0	0	0	0	126	24	8	8	8961	786	8.6102	1197
26-19.1462	194 414 1950	1462	27	18	15	13	12	6	3	6	0	0	0	0	0	126	27	8	9	1151	791	8.6143	2180
26-19.1063	191 416 1939	1063	24	12	24	24	0	0	13	0	2	0	0	0	0	125	24	9	10	8962	4949	8.6109	1312
26-19.1187	192 412 1932	1187	24	14	20	26	0	1	11	1	2	0	0	0	0	125	24	9	11	8963	5004	8.6115	1449
26-19.1460	194 412 1912	1460	24	18	12	30	0	1	12	0	1	0	0	0	1	125	24	10	12	8964	9542	8.6137	2037
26-19.1	152 568 1704	1	0	29	41	4	16	8	0	0	0	0	0	0	0	124	0	6	13	13068	1	8.5797	1
26-19.2	155 555 1720	2	5	20	45	5	13	10	0	0	0	0	0	0	0	124	5	6	14	12525	2	8.5819	2
26-19.9	164 536 1664	9	0	42	28	0	12	15	1	0	0	0	0	0	0	124	0	7	15	13073	12	8.5900	9

k = 26, Design generators

Design	Design Generators																									
26-19.1	7	11	19	29	37	41	49	55	59	77	78	87	91	97	98	111	116	120	125							
26-19.2	7	11	19	30	38	47	52	57	58	69	73	79	82	84	93	97	98	119	120							
26-19.3	7	11	13	19	21	31	38	41	55	59	70	73	87	91	99	101	106	116	120							
26-19.4	7	11	19	29	37	41	50	55	59	77	78	82	87	91	99	102	106	116	120							
26-19.5	7	11	19	29	35	38	41	44	50	55	69	73	92	95	100	103	113	120	125							
26-19.6	7	11	19	29	37	41	44	59	73	76	82	87	94	99	106	111	117	118	120							
26-19.7	7	11	19	29	35	38	41	44	50	55	69	73	82	92	95	100	103	120	125							
26-19.8	7	11	19	29	37	41	44	55	59	77	78	82	87	91	99	102	106	116	120							
26-19.9	7	11	19	29	35	46	53	57	60	69	73	76	82	87	100	109	118	120	123							
26-19.10	7	11	19	29	37	41	44	55	59	73	76	82	87	100	103	106	113	120	125							
26-19.11	7	11	19	29	35	38	41	44	50	55	69	73	82	92	95	97	100	120	125							
26-19.12	7	27	29	30	35	37	38	41	49	67	69	76	84	104	112	121	122	124	127							
26-19.13	7	11	19	30	38	41	49	50	52	77	78	82	84	91	97	108	119	120	126							
26-19.14	7	11	19	29	30	38	41	49	60	78	82	87	91	97	98	100	117	118	120							
26-19.15	7	11	19	29	37	41	44	50	55	59	73	76	87	100	103	106	113	120	125							
26-19.16	7	11	19	29	38	41	47	49	70	79	89	90	99	106	108	114	116	120	123							
26-19.17	7	11	14	19	35	37	38	41	52	59	69	70	89	90	95	103	106	117	120							
26-19.18	7	11	19	29	30	35	37	38	41	42	50	60	69	73	76	113	116	120	125							
26-19.49	7	11	19	29	35	37	38	41	44	50	55	69	73	92	95	100	113	120	125							
26-19.224	7	11	19	22	28	38	52	57	69	70	73	79	82	84	93	98	108	119	120							
26-19.935	7	11	13	21	26	47	51	54	78	81	100	104	107	109	112	121	122	124	127							
26-19.997	7	11	19	21	22	25	26	28	31	35	45	46	67	77	78	117	118	120	123							
26-19.1063	7	11	13	14	19	38	57	60	73	85	95	101	106	113	114	116	120	125	126							
26-19.1187	7	11	13	19	25	26	41	53	59	78	86	95	97	98	104	112	121	122	124							
26-19.1460	7	11	19	29	30	38	41	47	70	81	82	84	87	99	101	121	122	124	127							
26-19.1462	7	11	19	29	37	38	41	42	44	50	62	77	78	85	89	99	111	118	120							
26-19.1862	7	11	21	26	35	37	41	52	59	74	79	86	100	103	104	112	121	122	124							
26-19.2093	7	11	21	26	35	37	41	52	59	61	74	79	86	100	104	112	121	122	124							
26-19.2095a	7	11	21	26	35	37	41	52	56	74	79	86	100	103	104	112	121	122	124							
26-19.2098	7	11	21	26	35	37	41	52	59	61	79	86	100	103	104	112	121	122	124							
26-19.2612b	7	11	22	25	31	35	46	50	52	69	93	98	103	104	109	112	121	122	124							
26-19.13485	7	19	21	22	35	37	38	49	52	67	69	70	81	87	97	98	111	112	115							
26-19.13486	7	19	21	22	35	37	38	49	50	67	69	70	81	87	97	100	111	112	115							
26-19.13487	7	19	21	22	35	37	38	49	52	67	69	70	81	82	87	97	111	112	115							
26-19.13488	7	19	21	22	35	37	38	67	69	81	82	87	92	98	100	103	112	117	118							
26-19.13489	7	19	21	22	35	37	38	49	50	52	67	69	70	81	87	97	111	112	115							
26-19.13490	7	19	21	22	35	37	38	49	50	67	69	70	81	84	98	112	117	118	123							
26-19.13491	7	19	21	22	35	37	38	49	50	67	69	70	81	82	87	97	111	112	115							
26-19.13492	7	19	21	22	35	37	38	49	50	52	67	81	82	87	92	100	103	112	115							
26-19.13493	7	19	21	22	35	37	38	49	50	67	69	70	81	82	84	87	97	111	112							
26-19.13494	7	19	21	22	35	37	38	49	67	69	70	81	82	84	87	97	111	112	118							

k = 27, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)			alp										df		C2FI		Lmax		CD2*		CD2	
			wlp rank												rank	rank	rank	rank	rank				
27-20.1	180	690	2200	1	0	15	55	0	12	16	0	0	0	0	0	125	0	6	6	7696	1	7.7798	1
27-20.2	195	624	2304	2	0	30	12	39	3	2	9	0	1	0	0	123	0	9	19	7697	1326	7.7898	2
27-20.3	196	646	2152	3	0	29	41	0	4	18	6	0	0	0	0	125	0	7	7	7698	3	7.7920	5
27-20.4a	197	617	2296	4	0	33	9	35	9	0	7	3	0	0	0	123	0	8	20	7699	71	7.7912	3
27-20.4b	197	617	2296	4	0	30	18	26	12	0	7	3	0	0	0	123	0	8	20	7699	71	7.7912	3
27-20.6	200	610	2278	6	0	33	15	24	14	0	6	4	0	0	0	123	0	8	22	7701	73	7.7935	6
27-20.7	200	630	2172	7	5	20	45	0	7	11	10	0	0	0	0	125	5	7	8	7316	4	7.7948	8
27-20.8	201	610	2274	8	0	30	24	16	15	1	8	0	2	0	0	123	0	9	23	7702	1327	7.7946	7
27-20.9	202	588	2488	9	24	0	16	45	0	12	0	3	0	0	0	127	24	9	1	5505	1328	7.7950	9
27-20.10	203	603	2266	10	0	36	12	22	16	0	5	5	0	0	0	123	0	8	24	7703	74	7.7959	10
27-20.11	206	596	2248	11	3	30	18	17	18	0	4	6	0	0	0	123	3	8	25	7499	75	7.7983	13
27-20.12	207	592	2279	12	15	18	22	24	0	15	0	6	0	0	0	127	15	8	2	5934	76	7.7992	17
27-20.13	207	596	2244	13	0	42	0	29	15	0	7	0	3	0	0	123	0	9	26	7704	1329	7.7993	20
27-20.14	208	566	2488	14	20	2	33	6	28	6	0	0	2	1	0	125	20	10	9	5541	3728	7.7992	16
27-20.15	209	565	2384	15	8	22	16	28	10	2	3	7	0	0	0	123	8	8	27	6937	77	7.7993	18
27-20.16	210	546	2512	16	18	9	28	9	18	13	0	0	3	0	0	125	18	9	10	5651	1330	7.7993	19
27-20.17	210	548	2472	17	8	6	48	1	18	6	0	6	1	0	0	121	8	9	83	6938	1331	7.7994	21
27-20.18a	210	562	2314	18	0	3	29	42	6	0	0	3	4	0	0	114	0	9	6488	7705	1332	7.7995	24
27-20.18b	210	562	2314	18	0	3	27	48	0	2	0	3	4	0	0	114	0	9	6488	7705	1332	7.7995	24
27-20.20	210	563	2314	20	0	3	29	42	6	0	0	3	4	0	0	114	0	9	6490	7707	1334	7.7997	27

k = 27, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)			wlp rank			alp			df	C2FI	Lmax	df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2 rank					
27-20.9	202	588	2488	9	24	0	16	45	0	12	0	0	3	0	0	0	0	127	24	9	1	5505	1328	7.7950	9
27-20.12	207	592	2279	12	15	18	22	24	0	15	0	6	0	0	0	0	0	127	15	8	2	5934	76	7.7992	17
27-20.23	210	580	2416	23	24	0	16	51	0	0	6	3	0	0	0	0	0	127	24	9	3	5506	1337	7.8022	67
27-20.1023	234	484	2576	1023	24	18	16	15	0	18	0	6	3	0	0	0	0	127	24	9	4	5507	1425	7.8195	2110
27-20.1221	237	472	2543	1221	24	12	12	36	0	0	1	12	0	2	0	0	0	126	24	10	5	5508	4315	7.8212	2635
27-20.1	180	690	2200	1	0	15	55	0	12	16	0	0	0	0	0	0	0	125	0	6	6	7696	1	7.7798	1
27-20.3	196	646	2152	3	0	29	41	0	4	18	6	0	0	0	0	0	0	125	0	7	7	7698	3	7.7920	5
27-20.7	200	630	2172	7	5	20	45	0	7	11	10	0	0	0	0	0	0	125	5	7	8	7316	4	7.7948	8
27-20.14	208	566	2488	14	20	2	33	6	28	6	0	0	2	1	0	0	0	125	20	10	9	5541	3728	7.7992	16
27-20.16	210	546	2512	16	18	9	28	9	18	13	0	0	3	0	0	0	0	125	18	9	10	5651	1330	7.7993	19

k = 27, Design generators

Design	Design Generators																										
27-20.1	7	11	19	30	38	47	52	57	58	69	73	79	82	84	93	97	98	108	119	120							
27-20.2	7	11	19	29	30	38	41	49	60	78	82	87	91	97	98	100	109	117	118	120							
27-20.3	7	11	19	29	37	41	49	50	55	59	77	78	87	91	97	98	111	116	120	125							
27-20.4a	7	11	19	29	30	38	41	49	60	78	82	84	87	91	97	98	100	109	118	120							
27-20.4b	7	11	19	29	30	37	38	41	49	60	78	82	87	91	97	98	100	109	118	120							
27-20.6	7	11	19	29	37	41	44	59	69	73	76	82	87	94	99	106	111	117	118	120							
27-20.7	7	11	19	30	38	47	52	57	58	69	70	73	79	82	84	93	97	98	119	120							
27-20.8	7	11	19	29	37	41	44	50	55	59	62	73	76	85	86	91	99	102	120	125							
27-20.9	7	11	13	14	19	38	47	57	58	69	82	84	91	93	105	108	113	119	120	126							
27-20.10	7	11	19	29	37	41	44	59	73	76	82	87	91	94	99	106	111	117	118	120							
27-20.11	7	11	19	29	35	37	38	41	44	50	55	69	73	82	92	95	100	103	120	125							
27-20.12	7	11	19	30	38	47	52	57	69	70	73	79	82	84	93	97	98	110	119	120							
27-20.13	7	11	13	14	19	21	22	35	38	47	62	73	76	79	81	101	116	120	123	125							
27-20.14	7	11	19	29	30	41	50	63	77	86	88	101	102	104	107	112	115	121	122	124							
27-20.15	7	11	19	29	37	38	41	44	50	55	67	69	70	89	97	103	109	118	120	123							
27-20.16	7	27	29	30	35	37	38	41	49	67	69	74	76	79	104	112	121	122	124	127							
27-20.17	7	11	13	14	19	21	22	41	50	61	73	84	91	99	101	111	113	119	120	126							
27-20.18a	7	11	19	29	30	37	38	44	55	57	70	73	74	92	97	98	103	117	118	120							
27-20.18b	7	11	19	29	30	35	45	53	54	57	60	67	69	73	82	92	98	111	119	120							
27-20.20	7	11	19	29	30	37	38	44	52	57	70	73	74	92	97	98	103	117	118	120							
27-20.23	7	11	19	29	30	35	37	38	41	42	50	60	63	69	73	76	113	116	120	125							
27-20.1023	7	11	13	21	26	47	51	54	78	81	100	104	107	109	112	117	121	122	124	127							
27-20.1043	7	11	21	26	35	37	41	52	59	61	74	79	86	100	103	104	112	121	122	124							
27-20.1192	7	11	21	26	35	37	41	52	56	59	74	79	86	100	103	104	112	121	122	124							
27-20.1221	7	11	13	14	19	38	57	60	73	85	95	101	106	113	114	116	119	120	125	126							
27-20.1235	7	11	19	21	26	35	37	41	52	59	74	79	86	100	103	104	112	121	122	124							
27-20.1298a	7	11	21	26	35	37	41	52	59	74	79	86	88	100	103	104	112	121	122	124							
27-20.1298b	7	11	21	26	31	35	45	62	70	73	74	82	94	97	104	112	117	121	122	124							
27-20.1300	7	19	25	28	38	47	49	61	67	69	78	81	91	100	103	107	112	121	122	124							
27-20.1301	7	14	19	25	28	31	37	38	42	47	52	70	75	81	93	104	112	121	122	124							
27-20.8042	7	19	21	30	35	37	38	49	50	55	67	69	70	76	81	87	98	112	117	118							
27-20.8067	7	19	21	22	35	37	38	49	52	67	69	70	81	87	97	98	111	112	115	117							
27-20.8068	7	19	21	22	35	37	38	49	52	67	69	70	81	87	97	98	100	111	112	115							
27-20.8069	7	19	21	22	35	37	38	49	50	52	67	69	70	81	87	97	98	111	112	115							
27-20.8070	7	19	21	22	35	37	38	49	50	67	69	70	81	82	87	97	100	111	112	115							
27-20.8071	7	19	21	22	35	37	38	49	50	67	69	70	81	82	84	87	97	100	111	112							
27-20.8072	7	19	21	22	35	37	38	49	50	67	69	70	81	82	84	87	97	100	111	112							
27-20.8073	7	19	21	22	35	37	38	49	50	52	67	69	70	81	82	84	87	97	111	112							
27-20.8074	7	19	21	22	35	37	38	49	55	67	69	70	81	82	84	87	112	117	118	123							
27-20.8075	7	19	21	22	35	37	38	49	50	52	55	67	69	84	87	100	103	105	112	115							

Design	Design Generators																							
28-21.1	7	11	19	30	38	47	52	57	58	69	73	79	82	84	93	97	98	108	119	120	126			
28-21.2	7	11	19	30	38	47	52	57	58	69	70	73	79	82	84	93	97	98	108	119	120			
28-21.3	7	11	19	29	37	38	41	44	50	55	67	69	70	89	92	97	103	109	118	120	123			
28-21.4	7	11	19	29	37	41	44	50	55	59	62	73	76	85	86	91	99	102	106	120	125			
28-21.5	7	11	13	14	19	21	22	25	35	38	47	62	73	76	79	81	101	116	120	123	125			
28-21.6	7	11	13	14	19	21	22	38	41	50	52	59	73	79	84	93	99	101	108	119	120			
28-21.7	7	11	19	29	30	35	44	52	57	67	69	73	74	81	95	97	100	103	118	120	123			
28-21.8	7	11	13	14	19	21	22	38	41	50	52	59	70	73	84	93	99	101	108	119	120			
28-21.9a	7	11	13	19	22	26	28	38	41	47	50	73	79	82	84	93	99	106	108	120	126			
28-21.9b	7	11	19	29	30	35	38	41	50	60	77	78	86	89	90	95	97	116	119	120	126			
28-21.9c	7	11	19	29	30	38	41	42	49	50	60	67	69	70	76	90	97	103	117	118	120			
28-21.12	7	11	19	29	30	35	45	53	57	58	63	73	74	92	97	108	111	116	120	123	125			
28-21.13	7	11	19	29	35	45	46	53	54	57	60	67	69	70	76	84	89	90	100	120	126			
28-21.14	7	11	19	29	38	41	42	49	60	67	77	78	85	86	89	90	95	97	98	120	125			
28-21.15a	7	11	13	19	22	26	28	38	41	47	50	73	79	82	84	93	106	108	113	120	126			
28-21.15b	7	11	19	29	30	35	38	41	49	60	77	78	86	89	90	95	97	116	119	120	126			
28-21.15c	7	11	19	29	30	38	41	42	49	50	60	67	69	70	90	97	103	109	117	118	120			
28-21.18	7	11	19	29	30	38	41	44	70	74	81	82	101	104	107	112	115	121	122	124	127			
28-21.19	7	11	13	14	19	21	22	38	41	50	61	73	79	82	91	99	101	113	119	120	126			
28-21.58	7	11	19	25	26	28	31	35	45	46	53	54	59	69	73	76	79	84	113	120	125			
28-21.172	7	11	19	29	30	37	41	44	47	67	69	70	73	82	92	95	109	110	113	116	120			
28-21.681	7	11	21	26	35	37	41	52	59	61	74	79	86	88	100	103	104	112	121	122	124			
28-21.732	7	11	13	21	26	35	37	41	52	59	74	79	86	88	100	103	104	112	121	122	124			
28-21.733	7	11	19</																					

k = 29, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)	wlp rank	alp										df	C2FI	Lmax	rank	C2FI	Lmax	rank	CD2*	CD2 rank
29-22.1	266 945 3472	1	0	0	70	0	0	0	28	0	0	0	0	127	0	7	1	1914	1	6.4312	1
29-22.2	287 823 3819	2	0	30	0	30	21	4	1	0	10	0	0	125	0	9	4	1915	20	6.4414	2
29-22.3	289 810 3744	3	0	0	8	48	24	0	0	0	0	0	0	116	0	10	1792	1916	261	6.4415	3
29-22.4	290 810 3733	4	0	0	10	44	26	0	0	0	1	5	1	116	0	11	1793	1917	791	6.4423	5
29-22.5	290 810 3734	5	0	0	8	50	20	2	0	0	1	5	1	116	0	11	1794	1918	792	6.4423	6
29-22.6	290 816 3798	6	0	30	6	18	27	5	0	0	9	1	0	125	0	10	5	1919	262	6.4432	14
29-22.7a	291 808 3724	7	0	0	10	46	22	2	0	0	2	3	2	116	0	11	1795	1920	793	6.4430	9
29-22.7b	291 808 3724	7	0	0	8	52	16	4	0	0	2	3	2	116	0	11	1795	1920	793	6.4430	9
29-22.9a	291 810 3722	9	0	0	10	46	22	2	0	0	2	3	2	116	0	11	1797	1922	795	6.4431	11
29-22.9b	291 810 3722	9	0	0	12	40	28	0	0	0	2	3	2	116	0	11	1797	1922	795	6.4431	11
29-22.11	291 810 3723	11	0	0	10	46	28	0	0	0	2	3	2	116	0	11	1799	1924	797	6.4431	13
29-22.12a	291 812 3724	12	0	0	10	46	22	2	0	0	2	3	2	116	0	11	1800	1925	798	6.4433	15
29-22.12b	291 812 3724	12	0	0	8	52	16	4	0	0	2	3	2	116	0	11	1800	1925	798	6.4433	15
29-22.14a	292 808 3714	14	0	0	8	54	12	6	0	0	2	4	0	116	0	12	1803	1927	1266	6.4438	19
29-22.14b	292 808 3714	14	0	0	12	42	24	2	0	0	2	4	0	116	0	12	1802	1928	1265	6.4438	18
29-22.16a	292 810 3712	16	0	0	12	42	24	2	0	0	2	4	0	116	0	12	1804	1929	1267	6.4439	20
29-22.16b	292 810 3712	16	0	0	12	42	24	2	0	0	3	1	3	116	0	11	1804	1929	800	6.4439	20
29-22.18	292 810 3715	18	0	0	14	36	30	0	0	0	3	1	3	116	0	11	1806	1931	801	6.4440	22
29-22.19	292 812 3714	19	0	0	8	54	12	6	0	0	2	4	0	116	0	12	1807	1932	1268	6.4441	23

k = 29, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp										df	C2FI	Lmax	rank	C2FI	Lmax	rank	CD2*	CD2 rank
29-22.1	266 945 3472	1	0	0	70	0	0	0	28	0	0	0	0	127	0	7	1	1914	1	6.4312	1
29-22.114	306 729 4096	114	18	0	37	0	18	0	22	0	0	3	0	127	18	11	2	1407	834	6.4515	121
29-22.1725	370 537 4736	1725	27	0	40	0	0	0	16	0	9	0	0	127	27	11	3	1400	1146	6.4964	1878
29-22.2	287 823 3819	2	0	30	0	30	21	4	1	0	10	0	0	125	0	9	4	1915	20	6.4414	2
29-22.6	290 816 3798	6	0	30	6	18	27	5	0	0	9	1	0	125	0	10	5	1919	262	6.4432	14
29-22.147	309 740 3963	147	8	22	4	24	23	0	5	0	4	6	0	125	8	10	6	1661	265	6.4544	180
29-22.152	310 712 4156	152	12	15	18	3	24	11	1	9	0	0	3	125	12	11	7	1473	843	6.4540	163
29-22.181	312 704 4148	181	12	15	18	7	16	16	0	8	1	0	3	125	12	11	8	1474	850	6.4550	191
29-22.182	312 710 4134	182	12	15	18	7	16	16	0	8	1	0	3	125	12	11	9	1475	851	6.4554	205
29-22.224	315 700 4132	224	14	11	20	11	8	20	0	9	0	0	2	125	14	12	10	1444	1369	6.4572	267

k = 29, Design generators

Design	Design Generators																											
29-22.1	7	11	19	30	38	47	52	57	58	69	70	73	79	82	84	93	97	98	108	119	120	126						
29-22.2	7	11	19	29	35	37	38	57	63	69	70	73	79	81	87	97	98	103	109	117	120	123						
29-22.3	7	11	19	29	30	35	44	52	57	67	69	73	74	81	95	97	100	103	109	118	120	123						
29-22.4	7	11	19	29	38	41	42	49	60	67	77	78	85	86	89	90	95	97	98	108	120	125						
29-22.5	7	11	19	29	30	35	45	49	52	56	73	79	85	86	102	104	112	115	121	122	124	127						
29-22.6	7	11	19	29	37	41	44	50	55	59	62	73	76	85	86	91	99	102	106	111	120	125						
29-22.7a	7	11	19	29	30	35	41	44	47	54	56	67	77	78	81	84	88	104	112	121	122	124						
29-22.7b	7	11	13	14	19	28	35	44	53	57	58	67	76	85	89	90	102	105	120	123	125	126						
29-22.9a	7	11	19	29	30	35	45	46	49	52	56	73	79	85	102	104	112	115	121	122	124	127						
29-22.9b	7	11	19	29	30	35	44	52	55	57	67	69	73	74	95	97	100	103	109	118	120	123						
29-22.11	7	11	19	21	22	25	26	35	45	46	49	60	67	77	78	81	95	101	108	116	120	123						
29-22.12a	7	11	19	29	35	45	53	54	60	67	69	70	81	82	92	97	98	111	116	120	123	125						
29-22.12b	7	11	19	29	30	35	45	54	57	60	67	69	70	73	74	81	82	95	111	116	120	125						
29-22.14a	7	11	19	29	30	35	41	47	53	59	77	82	84	88	102	104	107	112	121	122	124	127						
29-22.14b	7	11	19	29	30	35	41	44	47	53	54	56	67	78	81	84	88	104	112	121	122	124						
29-22.16a	7	11	19	29	30	35	45	49	52	56	73	79	85	88	102	104	112	115	121	122	124	127						
29-22.16b	7	11	19	29	35	38	41	42	49	60	67	77	78	85	86	89	90	95	97	98	111	120						
29-22.18	7	11	19	29	35	45	53	54	60	67	69	70	73	81	82	92	97	98	111	116	120	123						
29-22.19	7	11	19	30	35	41	47	53	54	59	77	82	84	88	102	104	107	112	121	122	124	127						
29-22.114	7	11	13	19	21	35	38	57	60	67	69	70	73	76	81	84	93	98	103	110	118	120						
29-22.147	7	11	19	29	35	37	38	57	63	67	69	70	73	79	81	97	98	103	109	117	120	123						
29-22.152	7	11	13	19	25	26	35	38	41	42	52	67	69	73	74	87	100	103	109	114	120	123						
29-22.181	7	11	13	21	25	31	37	41	51	61	78	86	88	97	98	100	104	112	117	121	122	124						
29-22.182	7	11	13	21	25	28	31	37	41	51	61	78	86	88	97	98	104	112	117	121	122	124						
29-22.224	7	11	13	14	19	21	35	38	57	60	67	69	70	74	79	81	84	93	98	103	110	120						
29-22.379	7	11	19	21	26	28	35	37	41	52	59	62	70	77	87	98	104	112	117	121	122	124						
29-22.390	7	11	25	31	37	38	41	47	51	61	62	76	82	93	98	103	104	112	118	121	122	124						
29-22.405	7	11	19	21	26	28	35	41	52	56	59	62	73	79	86	91	97	100	112	121	122	124						
29-22.424	7	11	19	21	26	28	35	41	52	56	59	62	73	79	86	91	97	100	112	121	122	124						
29-22.432b	7	11	13	21	22	44	55	62	73	74	76	79	83	93	97	98	103	104	112	121	122	124						
29-22.434b	7	11	25	31	37	38	41	47	51	61	76	82	87	93	98	103	104	112	118	121	122	124						
29-22.1725	7	11	29	37	41	42	44	47	51	78	81	82	84	87	88	104	112	118	121	122	124	127						
29-22.1912	7	11	19	30	35	41	42	44	47	56	59	67	81	87	88	104	112	117	121	122	124	127						
29-22.1917	7	11	19	29	35	38	41	42	44	47	56	67	81	87	88	91	104	107	112	121	122	124	127					
29-22.1936	7	11	19	30	35	41	42	44	47	56	59	74	81	82	87	88	104	115	121	122	124	127						
29-22.2140	7	19	21	22	35	37	38	49	50	52	56	67	69	70	81	82	84	98	111	112	115	118						
29-22.2141	7	19	21	22	35	37	38	49	50	56	67	69	70	81	84	98	100	111	112	115	117	118						
29-22.2142	7	19	21	22	35	37	38	49	50	52	55	56	67	69	70	81	84	98	111	112	115	118						
29-22.2143	7	19	21	22	35	37	38	52	67	69	70	81	82	84	87	88	97	98	100	111	112	115						
29-22.2147	7	19	21	22	35	37	38	49	50	52	67	69	70	81	82	84	87	97	98	100	111	112						
29-22.2148	7	19	21	22	35	37	38	49	50	52	55	67	69	70	81	82	84	87	97	98	100	111	112					
29-22.2149	7	19	21	22	35	37	38	49	50	52	55	67	69	70	81	82	84	87	97	98	111	112						

k = 30, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ , ...)		alp										df		C2FI		Lmax		CD2*		CD2
		wlp rank													rank	rank			rank		
30-23.975	706	113	11548	975	57	0	0	0	0	0	0	0	0	25	6	0	0	1	862	6.1552	975
30-23.976	707	112	11536	976	57	0	0	0	0	0	0	0	0	28	0	3	0	2	961	6.1558	976
30-23.866	448	371	7098	866	36	0	0	0	0	21	27	0	0	0	7	0	0	3	558	5.9216	835
30-23.867	449	370	7086	867	36	0	0	0	0	24	21	3	0	0	7	0	0	4	559	5.9223	837
30-23.880	454	365	7096	880	36	0	0	0	0	6	16	20	6	0	6	1	0	5	821	5.9260	850
30-23.911	466	353	7148	911	36	0	0	5	3	16	16	3	5	0	4	3	0	6	829	5.9353	901
30-23.899	461	358	7138	899	35	0	0	0	12	12	9	15	0	0	6	1	0	7	827	5.9315	885

k = 30, Designs sorted based on minimizing Lmax

Design	wlp(w ₄ ,...)	wlp rank	alp										df	C2FI	Lmax	CD2*	CD2 rank						
30-23.245c	389 430 7378	245	29	0	0	0	15	16	13	18	0	0	0	0	0	121	29	8	146	129	1	5.8807	119
30-23.230	386 433 7404	230	29	0	0	2	5	22	26	4	3	0	0	0	0	121	29	9	141	125	2	5.8788	88
30-23.235	387 432 7396	235	29	0	0	1	9	21	18	12	1	0	0	0	0	121	29	9	141	126	3	5.8794	96
30-23.241	388 431 7386	241	29	0	0	1	12	15	21	12	1	0	0	0	0	121	29	9	145	128	4	5.8801	107
30-23.245a	389 430 7378	245	29	0	0	3	6	22	19	9	3	0	0	0	0	121	29	9	146	129	5	5.8807	119
30-23.255	390 429 7376	255	29	0	0	0	15	16	16	12	3	0	0	0	0	121	29	9	151	133	6	5.8815	132

k = 30, Design generators

Design	Design Generators																								
30-23.1	7	11	19	21	22	25	26	35	45	46	49	60	67	77	78	81	95	101	108	116	120	123	126		
30-23.2	7	11	19	29	30	35	45	46	49	52	56	73	79	85	88	102	104	112	115	121	122	124	127		
30-23.3	7	11	19	21	22	25	26	35	45	46	49	60	67	77	78	81	95	101	108	116	120	123	125		
30-23.4	7	11	19	29	30	35	41	44	47	53	54	56	67	77	78	81	84	88	104	112	121	122	124		
30-23.5	7	11	19	29	35	45	46	53	57	58	60	67	86	92	97	98	100	103	104	107	112	115	125		
30-23.6	7	11	19	29	30	35	45	54	57	60	67	69	70	73	74	81	82	95	97	111	116	120	125		
30-23.7	7	19	29	30	35	49	50	52	55	56	67	79	85	86	88	101	102	104	112	115	121	122	124		
30-23.8	7	11	19	29	35	45	46	53	57	58	60	63	67	86	97	98	100	103	104	107	112	115	125		
30-23.9	7	11	19	29	30	35	41	47	53	54	59	77	82	84	88	102	104	107	112	121	122	124	127		
30-23.10	7	11	19	30	35	41	47	53	54	59	78	82	84	88	101	102	104	107	112	121	122	124	127		
30-23.11	7	11	19	29	30	35	41	47	53	54	59	78	82	84	88	102	104	107	112	121	122	124	127		
30-23.12	7	11	13	14	19	21	22	38	41	47	50	52	73	79	82	84	91	99	101	106	113	120	126		
30-23.13	7	11	19	29	30	35	45	49	50	52	56	73	79	85	86	102	104	112	115	121	122	124	127		
30-23.14	7	11	19	29	30	35	41	47	53	59	78	82	84	88	101	102	104	107	112	121	122	124	127		
30-23.15	7	11	13	14	19	21	22	38	41	47	50	52	70	73	79	84	91	99	101	106	108	120	126		
30-23.16	7	11	19	29	30	35	41	47	53	56	59	77	82	84	88	102	104	107	112	121	122	124	127		
30-23.17	7	11	19	29	30	35	41	44	47	53	54	56	67	78	81	84	88	104	112	121	122	124	127		
30-23.18	7	11	19	21	22	25	26	35	45	46	49	60	67	77	78	81	95	101	105	108	116	120	123		
30-23.19	7	11	19	29	30	35	41	44	47	53	54	56	67	77	81	84	88	104	112	121	122	124	127		
30-23.20	7	11	19	30	35	41	47	53	54	56	59	77	82	84	88	102	104	107	112	121	122	124	127		
30-23.30	7	11	19	29	35	37	38	57	63	67	69	70	73	79	81	87	97	98	103	109	117	120	123		
30-23.126	7	19	21	30	35	37	38	44	49	58	67	69	73	81	84	95	98	100	103	104	112	117	126		
30-23.134	7	11	13	19	21	25	26	28	31	35	49	52	69	81	82	106	108	111	119	120	123	125	126		
30-23.135	7	11	13	19	21	25	26	28	31	35	38	49	67	69	82	106	108	111	119	120	123	125	126		
30-23.145	7	11	13	14	19	21	26	35	38	63	67	69	73	74	76	79	81	82	84	100	120	123	125		
30-23.156	7	11	19	29	35	37	38	57	63	67	69	70	73	79	81	97	98	100	103	109	117	120	123		
30-23.161	7	11	13	14	19	25	26	35	38	41	42	52	67	69	73	74	87	100	103	109	114	120	123		
30-23.230	7	11	13	14	21	26	31	35	37	41	52	56	59	69	79	86	97	103	104	112	121	122	124		
30-23.235	7	11	13	14	19	31	35	38	42	49	50	52	59	67	79	85	98	104	109	112	121	122	124		
30-23.239	7	11	14	25	26	28	31	45	53	67	70	85	88	97	98	100	103	104	112	121	122	124	127		
30-23.241	7	11	25	31	37	38	41	47	51	61	62	76	82	87	93	98	103	104	112	118	121	122	124		
30-23.245a	7	11	13	14	21	26	31	35	41	52	56	59	61	69	79	86	97	103	104	112	121	122	124		
30-23.245c	7	14	19	22	31	35	38	41	42	44	50	59	62	70	77	87	98	104	112	117	121	122	124		
30-23.255	7	11	13	14	21	26	28	31	35	37	41	52	59	69	79	86	97	103	104	112	121	122	124		
30-23.866	7	11	19	30	35	41	42	44	47	56	59	67	81	87	88	91	104	112	117	121	122	124	127		
30-23.867	7	11	19	30	35	41	42	44	47	56	59	67	81	87	88	104	112	115	117	121	122	124	127		
30-23.880	7	11	19	30	35	41	42	44	47	56	59	69	81	82	87	88	104	112	115	121	122	124	127		
30-23.899	7	11	19	30	35	37	41	42	44	47	56	67	81	82	87	88	104	112	115	121	122	124	127		
30-23.911	7	11	19	30	35	41	42	44	47	56	69	81	82	84	87	88	104	112	115	121	122	124	127		
30-23.975	7	19	21	22	35	37	38	49	50	52	55	67	69	70	81	82	84	87	97	98	100	111	112		
30-23.976	7	19	21	22	35	37	38	49	50	52	55	67	69	70	81	82	84	87	97	98	111	112	115		

k = 31, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	lmax	df	C2FI	lmax	CD2*	lmax	CD2
				rank	rank	rank	rank	rank	rank	rank	rank	rank
31-24.1	391 1134 5826	1	0 0 0 24 48 8	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.2	391 1134 5827	2	0 0 0 24 48 8	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.3	392 1132 5817	3	0 0 0 26 44 10	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.4	392 1134 5815	4	0 0 0 26 44 10	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.5	392 1136 5817	5	0 0 0 26 44 10	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.6	393 1132 5804	6	0 0 0 28 40 12	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.7	393 1136 5804	7	0 0 0 28 40 12	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.8	394 1132 5793	8	0 0 0 30 36 14	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.9	394 1136 5793	9	0 0 0 30 36 14	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.10	397 1128 5760	10	0 0 0 36 24 20	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.11	397 1136 5760	11	0 0 0 36 24 20	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.12	398 1102 5906	12	0 0 4 26 34 15	1 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.13	398 1103 5906	13	0 0 4 26 34 15	1 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.14a	399 1102 5894	14	0 0 6 22 36 15	1 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.14b	399 1102 5894	14	0 0 4 28 30 17	1 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.16	399 1103 5894	16	0 0 6 22 36 15	1 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.17	399 1104 5894	17	0 0 4 28 30 17	1 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0

k = 31, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp	df	C2FI	lmax	df	C2FI	lmax	CD2*	lmax	CD2
				rank	rank	rank	rank	rank	rank	rank	rank	rank
31-24.43	410 1060 6148	43	0 30 0 0 51 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.104	439 914 6688	104	12 9 24 3 0 33	0 3 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.86	434 952 6549	86	8 6 16 33 0 0	0 9 15 0 0	0 1 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.99	437 940 6576	99	8 6 16 33 0 0	0 16 2 6 0 0	0 1 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.105	439 938 6552	105	8 10 8 37 0 0	0 16 2 6 1 0	0 1 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.119	445 892 6772	119	11 6 22 10 14 0	0 9 15 0 0	0 4 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.125	449 880 6788	125	11 6 24 6 16 0	0 16 2 6 0 0	0 3 1 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.130	451 878 6768	130	13 2 28 2 18 0	0 16 2 6 0 0	0 4 0 0 0 0	0 2 1 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
31-24.37	408 848 7637	37	6 26 0 0 3 27	24 6 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0

k = 31, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w _i ,...)		wlp		alp		df		C2FI		Lmax		df		C2FI		Lmax		CD2*		CD2	
	rank		rank		rank		rank		rank		rank		rank		rank		rank		rank		rank	
31-24.433	819	126	14560	433	59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.390	525	420	8876	390	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.397	531	414	8896	397	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.401	539	406	8960	401	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.412	563	382	9184	412	35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.429	643	302	10672	429	35	2	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.422	591	354	9744	422	34	2	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.431	719	226	12176	431	34	26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

k = 31, Designs sorted based on minimizing Lmax

Design	wlp(w _i ,...)		wlp		alp		df		C2FI		Lmax		df		C2FI		Lmax		CD2*		CD2	
	rank		rank		rank		rank		rank		rank		rank		rank		rank		rank		rank	
31-24.128a	451	494	9208	128	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.135	453	492	9188	135	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31-24.121	447	498	9240	121	30	0	0	2	1	8	40	8	0	3	0	0	0	0	0	0	0	0
31-24.123	449	496	9224	123	30	0	0	1	4	13	25	14	4	1	0	0	0	0	0	0	0	0
31-24.128b	451	494	9208	128	30	0	0	3	0	16	24	15	1	3	0	0	0	0	0	0	0	0
31-24.142	455	490	9160	142	30	0	0	0	11	10	20	10	10	1	0	0	0	0	0	0	0	0
31-24.144	455	490	9172	144	30	0	0	2	5	17	14	16	6	2	0	0	0	0	0	0	0	0
31-24.145	455	490	9184	145	30	0	0	0	11	10	20	10	10	1	0	0	0	0	0	0	0	0

k = 31, Design generators

Design	Design Generators																																			
31-24.1	7	11	19	21	22	25	26	35	45	46	49	60	67	77	78	81	95	101	105	108	116	120	123	126												
31-24.2	7	11	19	29	30	35	41	44	47	53	54	56	67	77	78	81	84	88	104	112	121	122	124	127												
31-24.3	7	11	19	29	30	35	45	46	53	57	58	60	67	86	92	97	98	100	103	104	107	112	115	125												
31-24.4	7	11	19	29	30	35	41	47	53	54	56	59	77	82	84	88	102	104	107	112	121	122	124	127												
31-24.5	7	11	19	30	35	41	47	53	54	56	59	77	82	84	88	101	102	104	107	112	121	122	124	127												
31-24.6	7	11	19	29	30	35	45	46	53	57	58	60	67	86	95	97	98	100	103	104	107	112	115	125												
31-24.7	7	19	29	30	35	49	50	52	55	56	67	79	85	86	88	101	102	104	112	115	121	122	124	127												
31-24.8	7	11	19	29	30	35	45	46	53	57	58	60	63	67	86	97	98	100	103	104	107	112	115	125												
31-24.9	7	11	19	29	30	35	41	47	53	54	59	77	82	84	88	91	102	104	107	112	121	122	124	127												
31-24.10	7	11	19	29	30	35	41	47	53	59	78	82	84	88	91	101	102	104	107	112	121	122	124	127												
31-24.11	7	11	19	29	30	35	41	47	53	59	78	82	84	88	101	102	104	107	112	115	121	122	124	127												
31-24.12	7	11	19	29	30	35	41	42	47	53	56	59	77	82	84	88	102	104	107	112	121	122	124	127												
31-24.13	7	11	19	30	35	41	42	47	53	54	56	59	77	82	84	88	102	104	107	112	121	122	124	127												
31-24.14a	7	11	19	29	30	35	41	42	47	53	54	56	59	82	84	88	102	104	107	112	121	122	124	127												
31-24.14b	7	11	19	29	30	35	41	47	53	54	56	59	82	84	88	91	102	104	107	112	121	122	124	127												
31-24.16	7	11	19	29	30	35	41	42	44	47	53	54	56	67	77	81	84	88	104	112	121	122	124	127												
31-24.17	7	11	19	30	35	41	47	53	54	56	59	77	82	84	88	91	102	104	107	112	121	122	124	127												
31-24.37	7	11	19	29	30	35	37	38	41	42	49	67	69	70	76	84	104	107	112	115	121	122	124	127												
31-24.43	7	11	19	29	35	37	38	57	63	67	69	70	73	79	81	87	97	98	100	103	109	117	120	123												
31-24.86	7	11	13	14	19	21	22	26	35	38	63	67	69	73	74	76	79	81	82	84	100	120	123	125												
31-24.99	7	13	19	21	22	35	37	38	44	49	50	52	55	56	67	69	81	84	90	95	97	106	112	126												
31-24.104	7	11	14	25	26	28	31	45	53	56	67	70	85	88	97	98	100	103	104	112	121	122	124	127												
31-24.105	7	13	19	21	22	25	35	37	38	49	50	52	55	56	67	69	81	84	95	97	106	111	112	126												
31-24.119	7	11	19	25	35	41	42	44	54	56	59	67	77	78	81	88	104	107	112	115	121	122	124	127												
31-24.121	7	11	13	14	21	26	31	35	37	41	52	56	59	69	74	79	86	97	103	104	112	121	122	124												
31-24.123	7	11	13	14	19	28	31	35	38	42	49	50	52	59	67	76	85	98	104	109	112	121	122	124												
31-24.125	7	13	19	21	22	25	35	37	38	44	49	50	52	55	56	67	69	81	84	95	97	106	112	126												
31-24.128a	7	11	13	14	21	26	31	35	37	41	52	56	59	61	69	74	79	86	97	103	104	112	121	122	124											
31-24.128b	7	11	13	14	21	26	31	35	41	52	56	59	61	69	74	79	86	97	103	104	112	121	122	124												
31-24.130	7	13	19	21	22	25	35	37	38	41	49	50	52	55	56	67	69	81	84	95	97	111	112	126												
31-24.135	7	11	13	14	21	26	28	31	35	41	52	56	59	61	69	79	86	97	103	104	112	121	122	124												
31-24.142	7	11	19	21	22	25	31	35	38	47	49	56	59	61	67	78	82	84	98	103	112	121	122	124												
31-24.144	7	11	13	14	19	25	28	31	35	38	49	50	52	59	67	79	85	98	104	109	112	121	122	124												
31-24.145	7	11	13	14	19	28	31	35	38	42	49	50	52	59	62	67	85	98	104	109	112	121	122	124												
31-24.390	7	11	19	30	35	37	41	42	44	47	56	67	81	82	87	88	104	107	112	115	121	122	124	127												
31-24.397	7	11	19	30	35	41	42	44	47	56	59	69	81	82	84	87	88	104	112	115	121	122	124	127												
31-24.401	7	11	19	30	35	37	41	42	44	47	56	67	81	82	84	87	88	104	112	115	121	122	124	127												
31-24.412	7	11	19	30	35	37	41	42	44	47	56	81	82	84	87	88	91	104	112	115	121	122	124	127												
31-24.422	7	11	19	30	35	37	38	41	42	44	47	81	82	84	87	88	91	104	112	115	121	122	124	127												
31-24.429	7	19	21	22	35	37	38	49	50	52	56	67	69	70	81	82	88	97	98	111	112	115	117	118												
31-24.431	7	19	21	22	35	37	38	49	50	52	55	56	67	69	70	81	82	84	97	98	111	112	115	118												
31-24.433	7	19	21	22	35	37	38	49	50	52	55	67	69	70	81	82	84	87	97	98	100	111	112	115												

k = 32, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₄ ,...)	wlp rank	alp										df	C2FI Lmax	df rank	C2FI rank	Lmax rank	CD2* rank	CD2 rank
32-25.197	945 140 18200	197	61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32-25.178	609 476 11032	178	38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32-25.180	625 460 11160	180	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32-25.184	633 452 11640	184	34	0	4	5	19	0	0	0	0	0	0	0	0	0	0	0	0
32-25.189	681 404 12280	189	34	3	1	24	0	0	0	0	0	0	0	0	0	0	0	0	0
32-25.194	745 340 13432	194	34	3	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32-25.186	641 444 11576	186	33	4	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0
32-25.196	833 252 15288	196	33	28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32-25.89	529 556 11320	89	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32-25.123	545 540 11224	123	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

k = 32, Designs sorted based on minimizing Lmax

Design	wlp(w ₄ ,...)	wlp rank	alp										df	C2FI Lmax	df rank	C2FI rank	Lmax rank	CD2* rank	CD2 rank
32-25.75	521 564 11392	75	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32-25.79c	525 560 11336	79	31	0	0	0	6	10	15	15	10	6	0	0	0	0	0	0	0
32-25.82	525 560 11352	82	31	0	0	0	6	10	15	15	10	6	0	0	0	0	0	0	0
32-25.93	529 556 11344	93	31	0	0	0	9	7	15	15	7	9	0	0	0	0	0	0	0
32-25.71	517 568 11424	71	31	0	0	1	2	4	24	24	4	2	1	0	0	0	0	0	0
32-25.83	525 560 11360	83	31	0	0	2	1	13	15	15	13	1	2	0	0	0	0	0	0
32-25.91	529 556 11320	91	31	0	0	2	4	10	15	15	10	4	2	0	0	0	0	0	0
32-25.92	529 556 11344	92	31	0	0	3	0	16	12	12	16	0	3	0	0	0	0	0	0
32-25.96	533 552 11288	96	31	0	0	3	3	13	12	12	13	3	3	0	0	0	0	0	0
32-25.98	533 552 11312	98	31	0	0	2	7	7	15	15	7	7	2	0	0	0	0	0	0

k = 32, Design generators

Design	Design Generators																																	
32-25.1	7	11	19	29	30	35	45	46	53	54	57	60	67	86	89	95	97	98	100	103	104	107	112	115	125									
32-25.2	7	11	19	29	30	35	41	42	47	53	54	56	59	77	82	84	88	102	104	107	112	121	122	124	127									
32-25.3	7	11	19	30	35	41	42	47	53	54	56	59	77	82	84	88	101	102	104	107	112	121	122	124	127									
32-25.4	7	11	19	29	30	35	45	46	53	54	57	58	60	67	86	92	97	98	100	103	104	107	112	115	125									
32-25.5	7	11	19	29	30	35	41	47	53	54	56	59	77	82	84	88	91	102	104	107	112	121	122	124	127									
32-25.6	7	11	19	29	30	35	45	46	53	54	57	58	67	86	92	95	97	98	100	103	104	107	112	115	125									
32-25.7	7	11	19	29	30	35	45	46	53	54	57	58	60	63	67	86	97	98	100	103	104	107	112	115	125									
32-25.8	7	11	13	14	19	21	22	25	26	35	41	42	52	61	67	73	84	87	93	101	102	108	114	120	123									
32-25.9	7	11	13	14	19	21	22	25	26	35	41	42	52	61	67	73	84	87	93	101	102	108	111	114	120									
32-25.10	7	11	13	14	19	21	22	25	26	35	41	42	52	61	67	73	74	84	93	101	102	108	111	114	120									
32-25.11	7	11	13	14	19	21	22	25	26	35	41	42	52	61	67	73	84	88	102	104	107	112	121	122	124	127								
32-25.12	7	11	19	29	30	35	41	42	44	47	53	56	59	77	82	84	88	102	104	107	112	121	122	124	127									
32-25.13	7	11	19	30	35	41	42	44	47	53	54	56	59	77	82	84	88	102	104	107	112	121	122	124	127									
32-25.14	7	14	22	25	26	28	38	43	45	51	53	56	70	77	85	88	97	98	100	103	104	112	121	122	124									
32-25.15	7	11	19	29	30	35	45	46	53	54	57	58	67	77	86	92	97	98	100	103	104	107	112	115	125									
32-25.25	7	11	19	29	30	35	37	38	41	42	49	50	67	69	70	76	84	104	107	112	115	121	122	124	127									
32-25.42	7	11	19	29	38	41	42	47	49	56	62	70	73	82	87	88	101	104	107	112	115	121	122	124	127									
32-25.57	7	11	19	29	30	35	37	38	41	42	44	47	67	73	74	76	88	104	107	112	115	121	122	124	127									
32-25.60	7	11	19	25	26	28	31	35	37	38	41	42	44	67	69	70	73	74	76	109	110	117	118	120	123									
32-25.61	7	11	19	29	30	35	37	38	41	42	44	47	67	73	76	88	91	104	107	112	115	121	122	124	127									
32-25.64	7	11	19	25	31	35	46	50	52	56	59	74	76	86	88	97	103	104	107	112	115	121	122	124	127									
32-25.66	7	13	19	21	22	25	35	37	38	49	50	52	55	56	67	69	81	84	95	97	100	106	111	112	126									
32-25.71	7	11	13	14	19	28	31	35	38	42	49	50	52	59	67	76	79	85	98	104	109	112	121	122	124									
32-25.73	7	11	19	25	31	35	46	50	52	56	59	67	74	86	88	97	103	104	107	112	115	121	122	124	127									
32-25.75	7	11	13	14	19	25	28	31	35	38	42	49	50	52	55	56	67	79	85	98	104	109	112	121	122	124								
32-25.76	7	13	19	21	22	25	35	37	38	41	49	50	52	55	56	67	69	81	84	95	97	100	111	112	126									
32-25.79c	7	11	19	21	22	25	31	35	38	47	49	56	59	61	67	78	82	84	98	100	103	112	121	122	124									
32-25.82	7	11	13	14	19	25	28	31	35	38	49	50	52	59	62	67	79	85	98	104	109	112	121	122	124									
32-25.83	7	11	13	14	19	25	28	31	35	38	42	49	50	52	59	67	79	85	98	104	109	112	121	122	124									
32-25.91	7	19	22	29	35	37	38	41	42	44	50	67	69	73	76	82	91	104	107	112	115	121	122	124	127									
32-25.92	7	11	13	14	19	25	28	31	35	38	42	49	50	52	59	67	85	98	104	107	109	112	121	122	124									
32-25.93	7	11	13	14	19	25	28	31	35	38	42	49	50	52	59	62	67	79	85	98	104	112	121	122	124									
32-25.96	7	19	22	29	35	37	38	41	42	44	50	67	69	73	76	82	91	104	107	112	115	121	122	124	127									
32-25.98	7	11	19	30	35	38	41	42	44	47	67	69	74	76	81	87	88	104	109	112	117	121	122	124	127									
32-25.178	7	11	19	30	35	37	41	42	44	47	56	67	81	82	84	87	88	104	107	112	115	121	122	124	127									
32-25.180	7	11	19	30	35	37	41	42	44	47	56	67	81	82	84	87	88	91	104	112	115	121	122	124	127									
32-25.184	7	11	19	30	35	37	41	42	44	47	56	81	82	84	87	88	93	104	112	115	117	121	122	124	127									
32-25.186	7	11	19	30	35	37	41	42	44	47	56	81	82	84	87	88	91	93	104	112	115	121	122	124	127									
32-25.189	7	11	19	30	35	37	38	41	42	44	47	81	82	84	87	88	91	93	104	112	115	121	122	124	127									
32-25.194	7	19	21	22	35	37	38	49	50	52	56	67	69	70	81	82	84	88	97	98	100	111	112	115	118									
32-25.196	7	19	21	22	35	37	38	49	50	52	55	56	67	69	70	81	82	84	87	98	100	111	112	115	118									
32-25.197	7	19	21	22	35	37	38	49	50	52	55	56	67	69	70	81	82	84	87	97	98	100	111	112	115	117								

k = 33, Designs sorted based on degrees of freedom used

Design	wlp(w ₁ , ...)			wlp rank	alp										df C2FI Lmax		df rank	C2FI Lmax		rank	CD2*		rank	
33-26.38	592	648	14048	38	32	0	0	1	0	6	0	48	0	6	0	1	0	0	0	0	8	7	4.4900	21
33-26.39	592	1224	10272	39	8	6	0	49	0	0	0	0	24	0	0	1	0	6	0	2	49	38	4.5096	61
33-26.41	597	643	14008	41	32	0	0	0	3	0	28	0	28	0	3	0	0	0	0	3	9	3	4.4924	24
33-26.42c	600	640	13952	42	32	0	0	0	0	16	0	30	0	16	0	0	0	0	0	4	11	1	4.4937	27
33-26.42b	600	640	13952	42	32	0	0	4	0	0	0	54	0	0	0	4	0	0	0	4	11	8	4.4937	28
33-26.45	600	640	13984	45	32	0	0	0	0	16	0	30	0	16	0	0	0	0	0	6	13	2	4.4939	30
33-26.50	605	635	13928	50	33	0	0	0	0	0	30	0	30	0	0	0	1	0	0	7	5	64	4.4962	35
33-26.51b	605	635	13928	50	32	0	1	0	3	0	27	0	27	0	3	0	1	0	0	8	14	19	4.4962	35
33-26.51a	605	635	13928	50	32	0	0	0	0	6	0	25	0	25	0	6	0	0	0	8	14	4	4.4962	35
33-26.53	605	1147	10600	53	11	0	28	0	24	0	0	0	24	0	0	0	4	0	3	10	48	65	4.5138	69
33-26.54b	608	632	13920	54	32	1	0	0	0	13	0	34	0	13	0	0	0	0	1	11	16	39	4.4978	39
33-26.54a	608	632	13920	54	32	0	0	2	0	14	0	30	0	14	0	2	0	0	0	11	16	9	4.4978	39
33-26.56b	613	627	13912	56	32	0	0	0	9	0	22	0	22	0	9	0	0	0	0	13	18	5	4.5004	42
33-26.56a	613	627	13912	56	32	0	1	0	6	0	24	0	24	0	6	0	1	0	0	15	20	10	4.5016	44
33-26.58	616	624	13856	58	32	0	0	3	0	16	0	24	0	16	0	3	0	0	0	16	21	11	4.5018	45
33-26.59b	616	624	13888	59	32	0	0	3	0	16	0	24	0	16	0	3	0	0	0	16	21	11	4.5018	45
33-26.59a	616	624	13888	59	32	0	0	7	0	0	0	48	0	0	0	7	0	0	0	16	23	21	4.5042	50
33-26.61	621	619	13832	61	32	0	1	0	9	0	21	0	21	0	9	0	1	0	0	19	24	40	4.5055	52
33-26.62	624	616	13792	62	32	1	0	1	0	21	0	16	0	21	0	1	0	1	0	24	24	40	4.5055	52
33-26.63	624	616	13792	63	32	0	0	7	0	6	0	36	0	6	0	7	0	0	0	25	25	13	4.5055	55
33-26.64	624	616	13920	64	32	0	0	3	0	22	0	12	0	22	0	3	0	0	0	26	26	14	4.5063	59
33-26.66	629	611	13816	66	32	0	2	0	9	0	20	0	20	0	9	0	2	0	0	27	27	22	4.5083	59
33-26.68	632	608	13824	68	32	0	0	6	0	16	0	18	0	16	0	6	0	0	0	28	28	15	4.5100	62
33-26.69	632	608	13856	69	32	0	0	6	0	16	0	18	0	16	0	6	0	0	0	29	29	16	4.5102	63
33-26.71	637	603	13736	71	33	0	0	0	12	0	18	0	18	0	12	0	0	0	1	31	67	4.5121	66	
33-26.73	640	600	13792	73	32	1	0	6	0	13	0	22	0	13	0	6	0	1	0	32	31	41	4.5141	70
33-26.75	645	595	13720	75	32	0	7	0	0	0	24	0	24	0	0	0	7	0	0	27	32	23	4.5163	71
33-26.76	645	595	13976	76	32	0	0	0	21	0	10	0	10	0	21	0	0	0	0	28	33	6	4.5179	72
33-26.78	653	587	13896	78	32	0	3	0	15	0	13	0	13	0	15	0	3	0	0	29	34	24	4.5217	74
33-26.79	656	584	13792	79	32	2	0	9	0	4	0	32	0	4	0	9	0	2	0	30	35	42	4.5226	75
33-26.81	661	579	13880	81	32	0	6	0	9	0	16	0	16	0	9	0	6	0	0	31	36	25	4.5259	77
33-26.83	669	571	13800	83	33	0	6	0	6	0	18	0	18	0	6	0	6	0	1	32	7	68	4.5296	81
33-26.84	672	568	13664	84	32	7	0	0	0	7	0	34	0	7	0	0	7	0	0	33	38	43	4.5304	83
33-26.85	680	560	14112	85	32	0	15	0	15	0	16	0	0	16	0	15	0	0	0	34	39	17	4.5376	85
33-26.86	680	560	14144	86	32	0	0	15	0	16	0	0	0	16	0	15	0	0	0	35	40	18	4.5377	86
33-26.88	688	552	14048	88	32	3	0	9	0	19	0	0	0	19	0	9	0	3	0	36	41	44	4.5414	87
33-26.90	701	539	13608	90	39	0	0	0	0	0	24	0	24	0	0	0	0	7	0	37	2	70	4.5456	88
33-26.92	725	515	14520	92	32	0	10	0	21	0	0	0	0	0	21	0	10	0	0	38	43	26	4.5644	93
33-26.94	733	507	14440	94	35	0	4	0	24	0	0	0	0	0	24	0	4	0	3	39	3	71	4.5682	94
33-26.96	784	456	15328	96	32	6	0	25	0	0	0	0	0	0	25	0	6	0	0	45	45	45	4.6017	96
33-26.99	861	379	16744	99	35	0	28	0	0	0	0	0	0	0	0	28	0	3	0	41	4	72	4.6532	99

k = 33, Designs sorted based on degrees of freedom used (Continued)

Design	wlp(w ₁ ,...)	wlp rank	alp										df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2
													rank	rank	rank	rank	rank	rank	rank	rank
33-26.101	1085 155 22568	101	63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33-26.14	540 1120 11756	14	2 30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33-26.24	560 1080 11632	24	2 30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33-26.29	576 1048 11552	29	2 30	0	0	12 18	0	0	18 12	0	0	0	0	0	0	0	0	0	0	0

k = 33, Designs sorted based on the number of clear two-factor interactions

Design	wlp(w ₁ ,...)	wlp rank	alp										df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2
													rank	rank	rank	rank	rank	rank	rank	rank
33-26.101	1085 155 22568	101	63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33-26.90	701 539 13608	90	39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33-26.94	733 507 14440	94	35	0	4	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0
33-26.99	861 379 16744	99	35	0	28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33-26.50	605 635 13928	50	33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33-26.71	637 603 13736	71	33	0	0	12	0	18	0	12	0	0	0	0	0	0	0	0	0	0
33-26.83	669 571 13800	83	33	0	6	0	6	0	18	0	18	0	0	0	0	0	0	0	0	0
33-26.38	592 648 14048	38	32	0	0	1	0	6	0	48	0	6	0	0	0	0	0	0	0	0
33-26.41	597 643 14008	41	32	0	0	0	3	0	28	0	28	0	0	0	0	0	0	0	0	0

k = 33, Designs sorted based on minimizing Lmax

Design	wlp(w ₁ ,...)	wlp rank	alp										df	C2FI	Lmax	df	C2FI	Lmax	CD2*	CD2
													rank	rank	rank	rank	rank	rank	rank	rank
33-26.42c	600 640 13952	42	32	0	0	0	0	16	0	30	0	16	0	0	0	0	0	0	0	0
33-26.45	600 640 13984	45	32	0	0	0	0	16	0	30	0	16	0	0	0	0	0	0	0	0
33-26.41	597 643 14008	41	32	0	0	0	3	0	28	0	28	0	3	0	0	0	0	0	0	0
33-26.51a	605 635 13928	50	32	0	0	0	6	0	25	0	25	0	6	0	0	0	0	0	0	0
33-26.56b	613 627 13912	56	32	0	0	0	9	0	22	0	22	0	9	0	0	0	0	0	0	0
33-26.76	645 595 13976	76	32	0	0	0	21	0	10	0	10	0	21	0	0	0	0	0	0	0
33-26.38	592 648 14048	38	32	0	0	1	0	6	0	48	0	6	0	1	0	0	0	0	0	0
33-26.42b	600 640 13952	42	32	0	0	0	4	0	0	54	0	0	0	0	0	0	0	0	0	0
33-26.54a	608 632 13920	54	32	0	0	2	0	14	0	30	0	14	0	2	0	0	0	0	0	0
33-26.58	616 624 13856	58	32	0	0	0	3	0	16	0	24	0	16	0	3	0	0	0	0	0

k = 33, Design generators

Design	Design Generators																																	
33-26.1	7	11	19	29	30	35	45	46	53	54	57	58	60	67	77	86	92	97	98	100	103	104	107	112	115	125								
33-26.2	7	11	19	29	30	35	41	42	44	47	53	54	56	59	77	82	84	88	102	104	107	112	121	122	124	127								
33-26.3	7	11	19	29	30	35	45	46	53	54	57	58	60	67	86	92	95	97	98	100	103	104	107	112	115	125								
33-26.4	7	11	19	29	30	35	45	46	53	54	57	58	60	63	67	77	86	97	98	100	103	104	107	112	115	125								
33-26.5	7	11	13	14	19	21	22	25	26	28	35	41	42	52	61	67	73	84	87	93	101	102	108	111	114	120								
33-26.6	7	11	13	14	19	21	22	25	26	28	35	41	42	52	61	67	73	84	87	93	101	108	111	113	114	120								
33-26.7	7	11	13	14	19	21	22	25	26	28	35	41	42	52	61	67	73	74	84	93	101	102	108	111	114	120								
33-26.8	7	11	19	29	30	35	45	46	53	54	57	58	60	67	77	86	89	97	98	100	103	104	107	112	115	125								
33-26.9	7	11	19	29	30	35	45	46	53	54	57	58	60	67	77	86	90	97	98	100	103	104	107	112	115	125								
33-26.10	7	11	19	29	30	35	41	42	44	47	53	54	56	59	82	84	88	91	102	104	107	112	121	122	124	127								
33-26.11	7	11	13	14	19	21	22	25	26	28	35	41	42	52	61	67	73	74	84	93	101	108	113	114	120	123								
33-26.12	7	11	13	14	19	21	22	25	26	28	35	41	42	52	55	61	67	73	74	84	93	101	102	108	114	120								
33-26.13	7	11	13	14	19	21	22	25	26	28	35	41	42	52	55	61	67	73	74	84	93	101	108	113	114	120								
33-26.14	7	11	19	29	30	35	37	38	41	42	49	50	67	69	70	76	79	84	104	107	112	115	121	122	124	127								
33-26.15	7	13	19	21	22	25	28	35	37	38	44	49	50	52	55	56	69	75	78	81	84	95	97	112	123	126								
33-26.16a	7	13	19	21	22	25	28	35	37	38	41	49	50	52	55	56	69	75	78	81	84	95	97	106	112	126								
33-26.16b	7	13	19	21	22	25	28	35	37	38	44	49	50	52	55	56	69	75	78	81	84	95	97	106	112	126								
33-26.18	7	13	19	21	22	25	28	35	37	38	44	49	50	52	55	56	69	75	78	81	84	95	97	106	112	126								
33-26.19	7	13	19	21	22	25	28	35	37	38	41	49	50	52	55	56	69	75	78	81	84	90	95	97	112	126								
33-26.20	7	13	19	21	22	25	28	35	37	38	49	50	52	55	56	69	75	78	81	84	95	97	100	106	112	126								
33-26.24	7	11	19	29	38	41	42	47	49	56	62	70	73	82	87	88	94	101	104	107	112	115	121	122	124	127								
33-26.29	7	11	19	29	30	35	37	38	41	42	44	47	67	73	74	76	79	88	104	107	112	115	121	122	124	127								
33-26.38	7	11	13	14	19	28	31	35	38	42	49	50	52	59	67	76	79	85	98	104	107	109	112	121	122	124								
33-26.39	7	13	19	21	22	25	35	37	38	49	50	52	55	56	67	69	81	84	95	97	100	106	111	112	117	126								
33-26.41	7	11	13	14	19	25	28	31	35	38	42	49	50	52	59	67	79	85	98	104	107	109	112	121	122	124								
33-26.42a	7	11	19	30	38	41	44	49	52	59	61	70	74	79	82	87	91	93	104	107	112	115	121	122	124	127								
33-26.42b	7	11	21	22	31	35	38	41	56	59	67	77	81	84	87	94	97	98	103	104	112	115	121	122	124	127								
33-26.42c	7	11	19	21	22	25	31	35	38	47	49	56	59	61	67	78	82	84	98	100	103	107	112	121	122	124								
33-26.45	7	11	13	14	19	25	28	31	35	38	42	49	50	52	59	62	67	79	85	98	104	109	112	121	122	124								
33-26.50	7	11	21	25	28	31	35	50	52	56	61	69	76	86	88	91	97	103	104	109	112	115	121	122	124	127								
33-26.51a	7	19	22	29	35	37	38	41	42	44	50	67	69	73	76	82	87	91	104	107	112	115	121	122	124	127								
33-26.51b	7	11	13	14	19	25	28	31	35	38	49	50	52	56	59	67	79	85	98	104	107	112	121	122	124	127								
33-26.53	7	13	19	21	22	25	35	37	38	41	49	50	52	55	56	67	69	81	84	95	97	100	111	112	117	126								
33-26.54a	7	11	19	30	35	37	41	42	44	47	56	67	69	74	76	81	87	88	104	109	112	117	121	122	124	127								
33-26.54b	7	11	13	14	19	28	31	35	38	49	50	52	56	59	67	76	79	85	98	104	107	112	121	122	124	127								
33-26.56a	7	11	19	30	35	37	41	42	44	47	56	67	74	76	81	87	88	91	104	109	112	117	121	122	124	127								
33-26.56b	7	11	19	30	35	37	38	41	42	44	47	67	69	74	76	81	87	88	104	109	112	117	121	122	124	127								
33-26.58	7	19	21	22	25	26	31	35	38	45	49	67	69	70	73	74	82	87	91	94	97	98	112	117	121	124								
33-26.59a	7	11	19	30	35	41	42	44	47	56	59	67	74	76	81	87	88	91	104	109	112	117	121	122	124	127								
33-26.59b	7	19	21	22	25	26	31	35	38	45	49	67	69	73	74	81	82	87	91	94	97	98	107	112	117	121								
33-26.61	7	11	19	30	35	37	41	42	44	47	56	67	69	74	76	81	87	88	104	112	115	117	121	122	124	127								

k = 33, Design generators (Continued)

Design		Design Generators																									
33-26.62	7	19	22	29	35	37	38	41	42	44	47	67	73	74	76	82	88	91	104	107	112	115	121	122	124	127	
33-26.63	7	11	19	30	35	41	42	44	47	56	59	67	69	74	76	81	87	88	104	112	115	117	121	122	124	127	
33-26.64	7	11	19	30	35	37	38	41	42	44	47	67	74	76	81	87	88	91	104	109	112	117	121	122	124	127	
33-26.66	7	11	19	30	35	37	41	42	44	47	56	67	69	74	81	82	87	88	104	110	115	117	121	122	124	127	
33-26.68	7	11	13	21	25	28	31	35	41	59	69	76	86	88	97	98	100	103	104	107	110	112	115	121	122	124	
33-26.69	7	11	19	30	35	37	41	42	44	47	56	69	74	81	82	87	88	93	104	110	115	117	121	122	124	127	
33-26.71	7	11	13	14	19	21	22	26	35	37	38	49	50	56	59	67	69	70	81	82	88	91	111	112	115	118	
33-26.73	7	11	19	30	35	37	41	42	44	47	56	67	69	74	81	82	87	88	104	112	115	117	121	122	124	127	
33-26.75	7	11	19	30	35	37	41	42	44	47	56	67	69	74	81	82	87	88	104	107	112	115	121	122	124	127	
33-26.76	7	11	19	30	35	37	38	41	42	44	47	69	74	81	82	87	88	93	104	110	115	117	121	122	124	127	
33-26.78	7	11	19	30	35	37	41	42	44	47	56	69	74	81	82	87	88	93	104	112	115	117	121	122	124	127	
33-26.79	7	11	19	30	35	37	41	42	44	47	56	67	69	81	82	84	87	88	104	110	115	117	121	122	124	127	
33-26.81	7	11	19	30	35	37	41	42	44	47	56	69	81	82	84	87	88	93	104	110	115	117	121	122	124	127	
33-26.83	7	11	19	30	35	37	41	42	44	47	56	67	69	81	82	84	87	88	104	112	115	117	121	122	124	127	
33-26.84	7	11	19	30	35	37	41	42	44	47	56	67	69	81	82	84	87	88	104	107	112	115	121	122	124	127	
33-26.85	7	11	19	30	35	37	38	41	42	44	47	69	74	81	82	87	88	93	104	112	115	117	121	122	124	127	
33-26.86	7	11	19	30	35	37	38	41	42	44	47	69	81	82	84	87	88	93	104	110	115	117	121	122	124	127	
33-26.88	7	11	19	30	35	37	41	42	44	47	56	69	81	82	84	87	88	93	104	112	115	117	121	122	124	127	
33-26.90	7	11	19	30	35	37	41	42	44	47	56	67	81	82	84	87	88	91	104	107	112	115	121	122	124	127	
33-26.92	7	11	19	30	35	37	38	41	42	44	47	69	81	82	84	87	88	93	104	112	115	117	121	122	124	127	
33-26.94	7	11	19	30	35	37	41	42	44	47	56	81	82	84	87	88	91	93	104	112	115	117	121	122	124	127	
33-26.96	7	11	19	30	35	37	38	41	42	44	47	81	82	84	87	88	91	93	104	112	115	117	121	122	124	127	
33-26.99	7	19	21	22	35	37	38	49	50	52	56	67	69	70	81	82	84	88	97	98	100	111	112	115	117	118	
33-26.101	7	19	21	22	35	37	38	49	50	52	55	67	69	70	81	82	84	87	97	98	100	111	112	115	117	118	

k = 34, Designs sorted based on word length pattern

Design	wlp(w ₁ ,...)	wlp rank	alp										df	C2FI	Lmax	df rank	Lmax rank	CD2*	CD2 rank
34-27.1	589 1800 10788	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.2	589 1801 10788	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.3	597 1764 10882	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.4	598 1764 10868	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.5	605 1728 10978	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.6	605 1728 10979	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.7	606 1728 10964	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.8	607 1715 11046	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.9	608 1728 10936	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.10	615 1680 11146	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.11	616 1280 14432	11	0	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.12	616 1680 11132	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.13	634 1600 11412	13	0	1	5	2	48	0	0	0	0	0	0	0	0	0	0	0	0
34-27.14	636 1600 11384	14	0	0	4	0	3	49	0	0	0	0	0	0	0	0	0	0	0
34-27.15	637 1568 11578	15	0	0	8	24	0	7	33	8	0	0	0	0	0	0	0	0	0
34-27.16	638 1568 11564	16	0	0	8	24	0	7	33	8	0	0	0	0	0	0	0	0	0
34-27.17	645 1536 11691	17	0	0	8	24	0	24	0	24	0	0	0	0	0	0	0	0	0
34-27.18a	646 1536 11676	18	0	2	4	26	0	24	0	24	0	0	0	0	0	0	0	0	0
34-27.18b	646 1536 11676	18	0	0	8	24	0	24	0	24	0	0	0	0	0	0	0	0	0
34-27.20	648 1536 11648	20	0	4	0	28	0	24	0	24	0	0	0	0	0	0	0	0	0

k = 34, Designs sorted based on degrees of freedom used

Design	wlp(w ₁ ,...)	wlp rank	alp										df	C2FI	Lmax	df rank	Lmax rank	CD2*	CD2 rank
34-27.11	616 1280 14432	11	0	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.21	656 1200 14184	21	0	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.23	680 1152 14240	23	0	32	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.26	720 1072 14504	26	0	38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.31	976 560 19880	31	0	62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.22	674 1424 12740	22	0	0	16	44	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.24	680 1408 12704	24	0	8	0	52	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.27	730 1200 13972	27	0	3	33	24	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.29	794 1008 15316	29	0	3	57	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-27.30	808 896 15904	30	0	32	0	28	0	0	0	0	0	0	0	0	0	0	0	0	0

k = 34, Designs sorted based on minimizing lmax

Design	wlp(w ₁ ,...)	wlp rank	alp										df	C2FI	lmax	df rank	lmax rank	CD2*	CD2 rank		
34-27.1	589 1800 10788	1	0	0	0	0	24	50	6	0	0	0	0	0	0	121	0	15	1	4.1085	2
34-27.2	589 1801 10788	2	0	0	0	0	24	50	6	0	0	0	0	0	0	121	0	15	2	4.1086	3
34-27.3	597 1764 10882	3	0	0	0	4	28	31	17	0	0	0	0	0	0	121	0	15	3	4.1111	4
34-27.5	605 1728 10978	5	0	0	0	12	12	48	0	8	0	0	0	0	0	121	0	15	4	4.1138	6
34-27.6	605 1728 10979	6	0	0	0	12	12	48	0	8	0	0	0	0	0	121	0	15	5	4.1138	7
34-27.8	607 1715 11046	8	0	0	0	15	17	21	27	0	0	0	0	0	0	121	0	15	6	4.1144	9
34-27.10	615 1680 11146	10	0	0	2	11	25	18	16	8	0	0	0	0	0	121	0	15	7	4.1171	11
34-27.15	637 1568 11578	15	0	0	8	24	0	7	33	8	0	0	0	0	0	121	0	15	8	4.1242	15
34-27.17	645 1536 11691	17	0	0	8	24	0	24	0	24	0	0	0	0	0	121	0	15	9	4.1272	18
34-27.4	598 1764 10868	4	0	0	0	6	24	33	17	0	0	0	0	0	0	121	0	16	10	4.1116	5

k = 34, Design generators

Design	Design Generators																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
34-27.1	7	11	19	29	30	35	45	46	53	54	57	58	60	67	77	86	89	92	97	98	100	103	104	107	112	115	125																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																						

k = 35, Designs sorted based on word length pattern

Design	wlp(w ₄ ,...)	wlp rank	alp										df	C2FI	lmax	df rank	lmax rank	CD2*	CD2 rank
35-28.1	665 2100 13020	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
35-28.2	665 2101 13020	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
35-28.3	674 2058 13140	3	0	0	0	0	18	35	27	0	0	0	0	0	0	0	0	0	3
35-28.4	683 2016 13263	4	0	0	0	4	16	36	16	8	0	0	0	0	0	0	0	0	4
35-28.5	684 2016 13248	5	0	0	0	0	6	12	38	16	8	0	0	0	0	0	0	0	5
35-28.6	694 1960 13468	6	0	0	0	10	22	7	33	8	0	0	0	0	0	0	0	0	6
35-28.7	703 1920 13599	7	0	0	0	2	6	24	24	0	24	0	0	0	0	0	0	0	7
35-28.8	704 1920 13584	8	0	0	4	2	26	24	0	24	0	0	0	0	0	0	0	0	8
35-28.9	727 1792 14127	9	0	0	4	4	28	0	0	24	24	0	0	0	0	0	0	0	9
35-28.10	728 1792 14112	10	0	0	4	28	0	0	24	24	0	0	0	0	0	0	0	0	10
35-28.11	776 1536 15264	11	0	0	32	0	0	0	0	48	0	0	0	0	0	0	0	0	11
35-28.12	776 1600 15712	12	0	0	8	52	0	0	0	0	0	24	4	0	0	0	0	0	12
35-28.13	840 1344 17248	13	0	0	32	28	0	0	0	0	0	0	0	0	0	0	0	0	13

k = 35, Designs sorted based on degrees of freedom used

Design	wlp(w ₄ ,...)	wlp rank	alp										df	C2FI	lmax	df rank	lmax rank	CD2*	CD2 rank
35-28.12	776 1600 15712	12	0	0	8	52	0	0	0	0	0	0	0	0	0	0	0	0	1
35-28.13	840 1344 17248	13	0	0	32	28	0	0	0	0	0	0	0	0	0	0	0	0	2
35-28.1	665 2100 13020	1	0	0	0	0	0	70	10	0	0	0	0	0	0	0	0	0	3
35-28.2	665 2101 13020	2	0	0	0	0	0	0	70	10	0	0	0	0	0	0	0	0	4
35-28.3	674 2058 13140	3	0	0	0	0	18	35	27	0	0	0	0	0	0	0	0	0	5
35-28.4	683 2016 13263	4	0	0	0	4	16	36	16	8	0	0	0	0	0	0	0	0	6
35-28.5	684 2016 13248	5	0	0	0	0	6	12	38	16	8	0	0	0	0	0	0	0	7
35-28.6	694 1960 13468	6	0	0	0	10	22	7	33	8	0	0	0	0	0	0	0	0	8
35-28.7	703 1920 13599	7	0	0	0	2	6	24	24	0	24	0	0	0	0	0	0	0	9
35-28.8	704 1920 13584	8	0	0	4	2	26	24	0	24	0	0	0	0	0	0	0	0	10
35-28.9	727 1792 14127	9	0	0	4	28	0	0	24	24	0	0	0	0	0	0	0	0	11
35-28.10	728 1792 14112	10	0	0	4	28	0	0	24	24	0	0	0	0	0	0	0	0	12
35-28.11	776 1536 15264	11	0	0	32	0	0	0	0	48	0	0	0	0	0	0	0	0	13
35-28.12	776 1600 15712	12	0	0	8	52	0	0	0	0	0	24	4	0	0	0	0	0	1
35-28.13	840 1344 17248	13	0	0	32	28	0	0	0	0	0	0	0	0	0	0	0	0	2

k = 35, Designs sorted based on minimizing Lmax

Design	wlp(w ₁ ,...)	wlp rank	alp			df			Lmax			Lmax			CD2*			CD2		
35-28.1	665 2100 13020	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.2	665 2101 13020	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.3	674 2058 13140	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.4	683 2016 13263	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.6	694 1960 13468	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.7	703 1920 13599	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.9	727 1792 14127	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.5	684 2016 13248	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.8	704 1920 13584	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35-28.10	728 1792 14112	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

k = 35, Design generators

Design		Design Generators																			
35-28.1	15 23 25 26 28 39 43 45 46 51 53 54 56 71 73 74 76 81 82 84 88 99 101 104 111 112 119 123 126																				
35-28.2	15 23 25 26 28 39 43 45 46 51 53 54 56 71 73 74 76 81 82 84 88 101 104 111 112 119 123 126																				
35-28.3	15 23 25 26 28 39 43 45 46 51 53 54 56 63 71 73 74 76 81 82 84 88 101 104 111 112 119 123 126																				
35-28.4	15 23 25 26 28 39 43 45 46 51 53 54 56 63 71 73 76 81 82 84 88 95 101 104 111 112 119 123 126																				
35-28.5	15 23 25 26 28 39 43 45 46 51 53 54 56 63 71 74 81 82 84 88 95 101 104 112 119 123 125 126																				
35-28.6	15 23 25 26 28 39 43 45 46 51 54 56 63 71 73 76 81 82 84 88 95 99 101 104 112 119 123 126																				
35-28.7	15 23 25 26 28 39 43 45 46 51 54 56 63 71 73 76 81 82 84 88 95 101 102 104 111 112 119 126																				
35-28.8	15 23 25 26 28 39 43 45 46 51 53 54 56 63 71 73 76 81 82 84 88 95 101 104 111 112 119 125 126																				
35-28.9	15 23 27 29 30 41 42 44 51 53 54 56 63 67 69 73 86 90 92 95 97 104 107 109 114 116 121 126																				
35-28.10	15 23 27 29 30 41 42 44 51 53 54 63 67 69 73 86 90 92 95 97 102 104 107 109 114 116 121 126																				
35-28.11	15 23 27 29 30 37 43 44 51 52 58 63 69 70 75 76 83 84 90 95 97 100 104 111 112 119 123 126																				
35-28.12	15 23 25 30 39 41 46 51 53 54 56 63 71 73 78 83 85 86 88 95 97 98 100 104 111 112 121 126																				
35-28.13	15 23 25 30 39 41 42 44 49 54 56 63 71 75 77 78 81 86 88 95 99 102 104 111 112 119 121 126																				

Design	wlp(w ₁ ,...)	wlp rank	alp		df	C2FI	Lmax	df rank	C2FI rank	Lmax rank	CD2*	CD2 rank				
37-30.1	854 2744 18886	1	0	0	0	0	21	51	8	0	0	1	1	3.2166	1	
37-30.2	865 2688 19080	2	0	0	0	0	6	26	24	0	0	0	2	2	3.2191	2
37-30.3	889 2560 19584	3	0	0	0	0	0	32	0	0	48	0	0	3	3.2246	3

Design	Design Generators																													
37-30.1	15	23	25	26	28	39	43	45	46	51	53	54	56	63	71	73	74	76	81	82	84	88	95	99	101	104	111	112	119	126
37-30.2	15	23	25	26	28	39	43	45	46	51	53	54	56	63	71	73	74	76	81	82	84	88	95	101	102	104	111	112	119	126
37-30.3	15	23	25	26	28	39	43	45	46	51	54	56	63	71	73	76	81	82	84	88	95	99	101	102	104	111	112	119	123	126

Design	wlp (w_4, \dots)	wlp rank	alp	df	C2FI	lmax rank	df rank	lmax rank	CD2*	CD2 rank
38-31.1	959 3136 22512	1	0 0 0 0 7 49 24	0 0 0 0 0 0 0	7 0	125 0	18 1	2.9795	1	
38-31.2	971 3072 22752	2	0 0 0 0 32 0 48	0 0 0 0 0 0 0	6 1	125 0	19 2	2.9819	2	

Design	Design Generators																														
88-31.1	15	23	25	26	28	39	43	45	46	51	53	54	56	63	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	126
88-31.2	15	23	25	26	28	39	43	45	46	51	53	54	56	63	71	73	74	76	81	82	84	88	95	101	102	104	111	112	119	125	126

k = 39, Designs sorted based on word length pattern

[illegible]

k = 39, Design generators

Design	Design Generators																															
39-32.1	15	23	25	26	28	39	43	45	46	51	53	54	56	63	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	126

k = 40, Designs sorted based on word length pattern

Design	wlp(w_4, \dots)	wlp rank	alp											df	C2FI	lmax rank	df	lmax rank	CD2*	CD2 rank				
40-33.1	1190	4096	31360	1	0	0	0	0	0	0	0	0	0	0	0	0	0	7	127	0	20	1	2.5767	1

k = 40, Design generators

Design	Design Generators																																
40-33.1	15	23	25	26	28	39	43	45	46	51	53	54	56	63	71	73	74	76	81	82	84	88	95	99	101	102	104	111	112	119	123	125	126

Vita

Robert M. Block is a 1987 National Merit Scholar. He graduated with Military Distinction from the United States Air Force Academy with a Bachelor of Science in Operations Research. He earned a Master of Science in Operations Research from the Industrial and Systems Engineering College at Georgia Tech. He received his Doctorate in Business Administration with a concentration in Statistics from the University of Tennessee, Knoxville.

Rob has experience as a Logistics Operations Research Analyst, and as a Financial Analyst. He has worked as a Logistics Research Analyst for Air Force Materiel Command Headquarters in Dayton, Ohio, as the Chief of Financial Analysis for the 39th Wing, Incirlik AB, Turkey, and as an Assistant Professor and Course Director in the Math Department at the United States Air Force Academy. He has been a command briefer for Air Force Materiel Command, and a Technical Editor for the Air Force Scientific Advisory Board.

Rob is a Distinguished Graduate from the Air Force Financial Management (Analysis) Officer Course, a Chief of Staff Award Winner at Squadron Officer School, and was named the 1997 USAFE Financial Analysis Officer of the Year. He was awarded the 1998 Distinguished Performance in Budgeting from the American Society of Military Comptrollers. He was honored as the 1999 Company Grade Officer of the Year for the Academy Math Department. He has also received the University of Tennessee's 2003 Provost award for Extraordinary Professional Promise. He has been awarded the Air Force Meritorious Service Medal with two oak leaf clusters.